Matlab and Simulink for Control

Automatica I (Laboratorio)

Matlab and Simulink

CACSD

Matlab and Simulink for Control

- Matlab introduction
- Simulink introduction
- Control Issues Recall
- Matlab design Example
- Simulink design Example

Part I

Introduction

What is MATLAB

- High-Performance language for technical computing
- Integrates computation, visualisation and programming
- MATLAB = *MATrix LABoratory*
- Features family of add-on, application-specific *toolboxes*

What are MATLAB components?

- Development Environment
- The MATLAB Mathematical Function Library
- The MATLAB language
- Graphics
- The MATLAB Application Program Interface

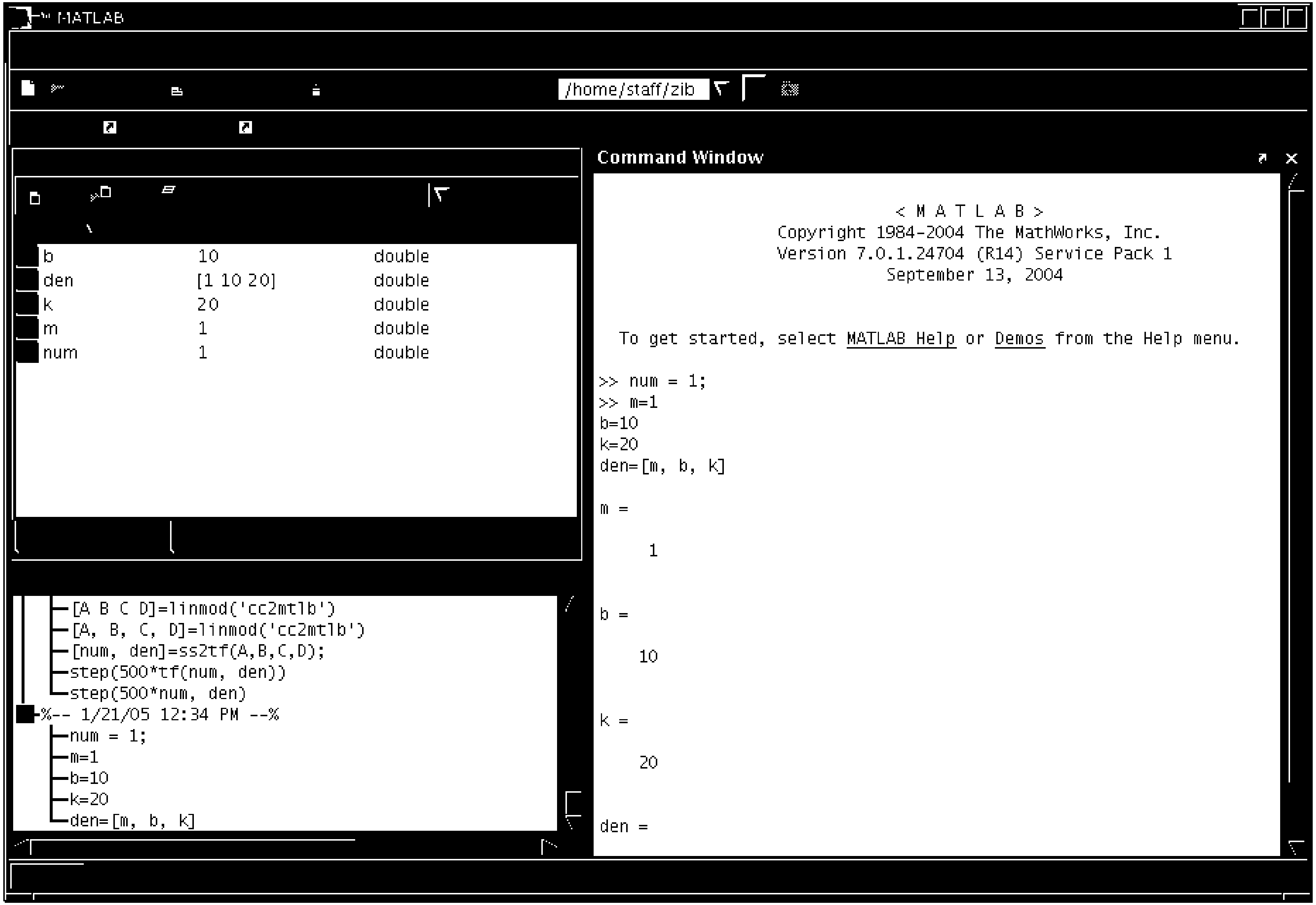
What is Simulink?

- Software Package for modelling, simulating and analysing dynamic systems
- Supports linear & Non-linear systems
- Supports continuous or discrete time systems
- Supports multirate systems
- Allows you to model real-life situations
- Allows for a top-down and bottom-up approach

How Simulink Works?

1. Create a block diagram (model)
2. Simulate the system represented by a block diagram

MATLAB Environment



The MATLAB Language

Dürer's Matrix

```
A=[16 3 2 13; 5 10 11 8; 9 6 7 12;4 15 14 1];
sum(A) %ans = 34 34 34 34
sum(A') %ans = 34 34 34 34
sum(diag(A)) %ans = 34
```

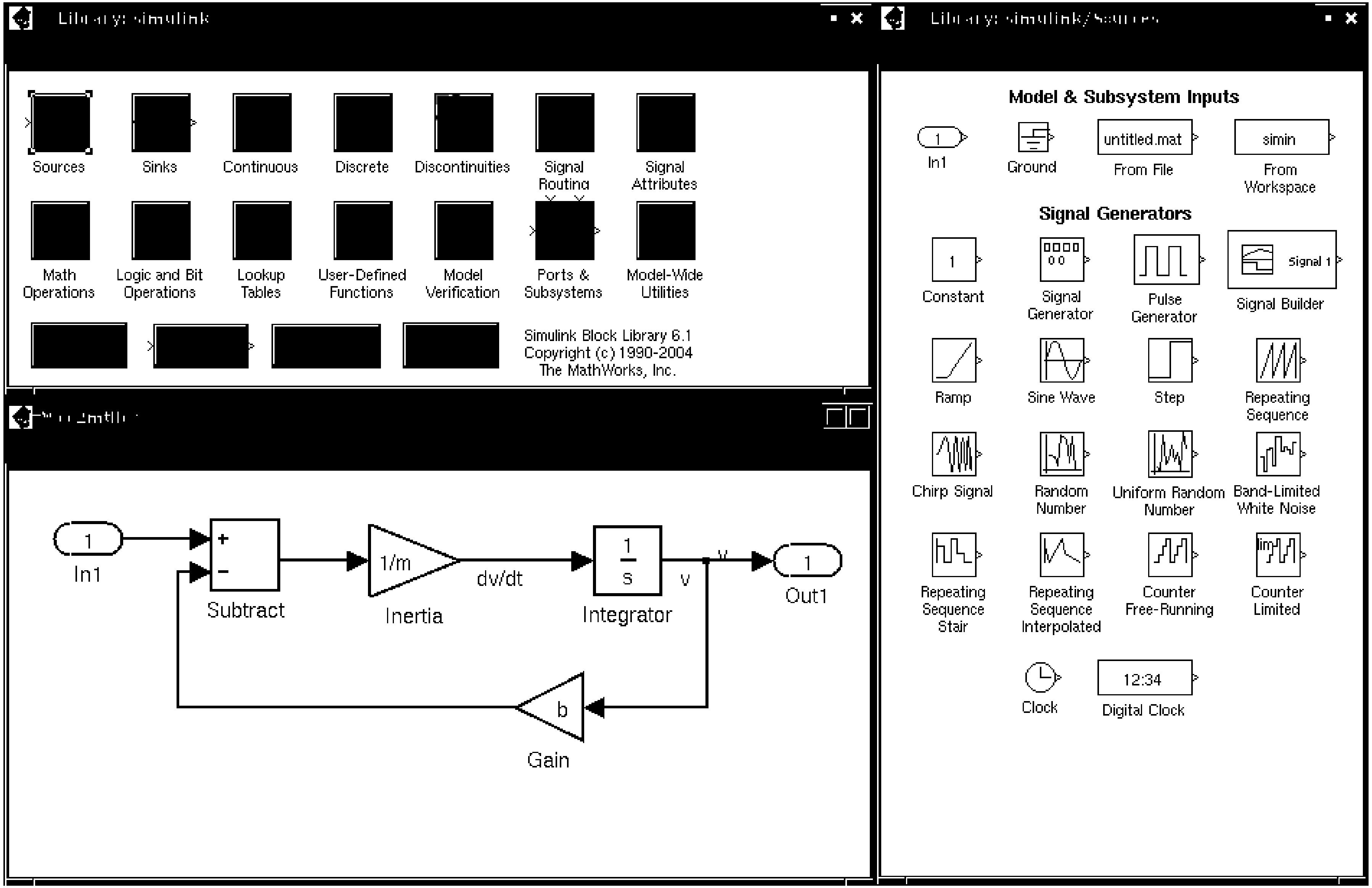
Operators

```
100:-7:50 % 100 93 86 79 72 65 58 51
sum(A(1:4,4)) % ans = 34
```

The MATLAB API

- You can use C or FORTRAN
- Pipes on UNIX, COM on Windows
- You can call MATLAB routines from C/FORTRAN programs and vice versa
- You can call Java from MATLAB

Simulink Environment



Starting Simulink

Just type in MATLAB

```
simulink
```

Part II

MATLAB – Background

Laplace Transform

Definition

The Laplace Transform is an integral transform perhaps second only to the Fourier transform in its utility in solving physical problems. The Laplace transform is defined by:

$$\mathcal{L}\left[f(t)\right](s) \equiv \int_0^\infty f(t)e^{-st}dt$$

Source: [1, Abramowitz and Stegun 1972]

Laplace Transform

Several Laplace Transforms and properties

f	$\mathcal{L}[f(t)](s)$	range
1	$\frac{1}{s}$	$s > 0$
t	$\frac{1}{s^2}$	$s > 0$
t^n	$\frac{n!}{s^{n+1}}$	$n \in Z > 0$
e^{at}	$\frac{1}{s-a}$	$s > a$

$$\mathcal{L}_t\left[f^{(n)}(t)\right](s) = s^n \mathcal{L}_t[f(t)] - \\ -s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0) \quad (1)$$

This property can be used to transform differential equations into algebraic ones. This is called Heaviside calculus

Heaviside Calculus Example

Let us apply the Laplace transform to the following equation:

$$f''(t) + a_1 f'(t) + a_0 f(t) = 0$$

which should give us:

$$\begin{aligned} \left\{s^2 \mathcal{L}_t\left[f(t)\right](s) - sf(0) - f'(0)\right\} + \\ + a_1 \left\{s \mathcal{L}_t\left[f(t)\right](s) - f(0)\right\} + \\ + a_0 \mathcal{L}_t\left[f(t)\right](s) = 0 \end{aligned}$$

which can be rearranged to:

$$\mathcal{L}_t\left[f(t)\right](s) = \frac{sf(0) + f'(0) + a_1 f(0)}{s^2 + a_1 s + a_0}$$

Transfer Functions

- For MATLAB modelling we need Transfer Functions
- To find the Transfer Function of a given system we need to take the Laplace transform of the system modelling equations (2) & (3)

System modelling equations

$$F = m\dot{v} + bv \tag{2}$$

$$y = v \tag{3}$$

Laplace Transform:

$$F(s) = msV(s) + bV(s)$$

$$Y(s) = V(s)$$

Transfer Functions – cntd.

- Assuming that our output is velocity we can substitute it from equation (5)

Transfer Function
Laplace Transform:

$$F(s) = msV(s) + bV(s) \quad (4)$$

$$Y(s) = V(s) \quad (5)$$

Transfer Function:

$$\frac{Y(s)}{F(s)} = \frac{1}{ms + b} \quad (6)$$

Matlab Functions – Transfer Function I

What is tf?

Specifies a SISO transfer function for model $h(s) = n(s)/d(s)$

```
h = tf(num, den)
```

What are num & den?

row vectors listing the coefficients of the polynomials $n(s)$ and $d(s)$ ordered in descending powers of s

Source: MATLAB Help

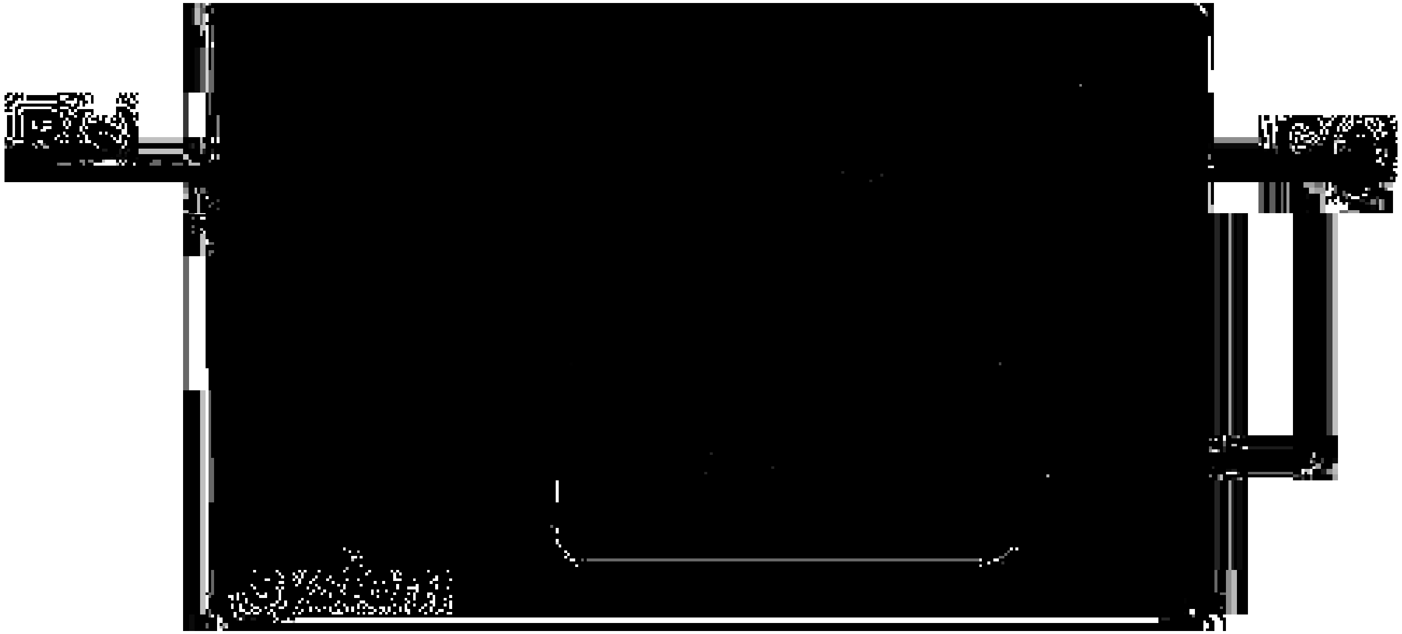
Matlab Functions – Transfer Function II

`tf` Example

$$T(s) = \frac{2s-3}{s+1} \equiv \texttt{h=tf([2 -3], [1 1])}$$

$$T(s) = \frac{2s+1}{4s^2+s+1} \equiv \texttt{h=tf([2 1], [4 1 1])}$$

Matlab Functions – Feedback I



Matlab code

```
sys = feedback(forward,backward);
```

Source: [2, Angermann et al. 2004]

MATLAB Functions – Feedback II

- obtains a closed-loop transfer function directly from the open-loop transfer function
- no need to compute by hand

Example

$$Forward = \frac{1}{sT_i} \tag{7}$$

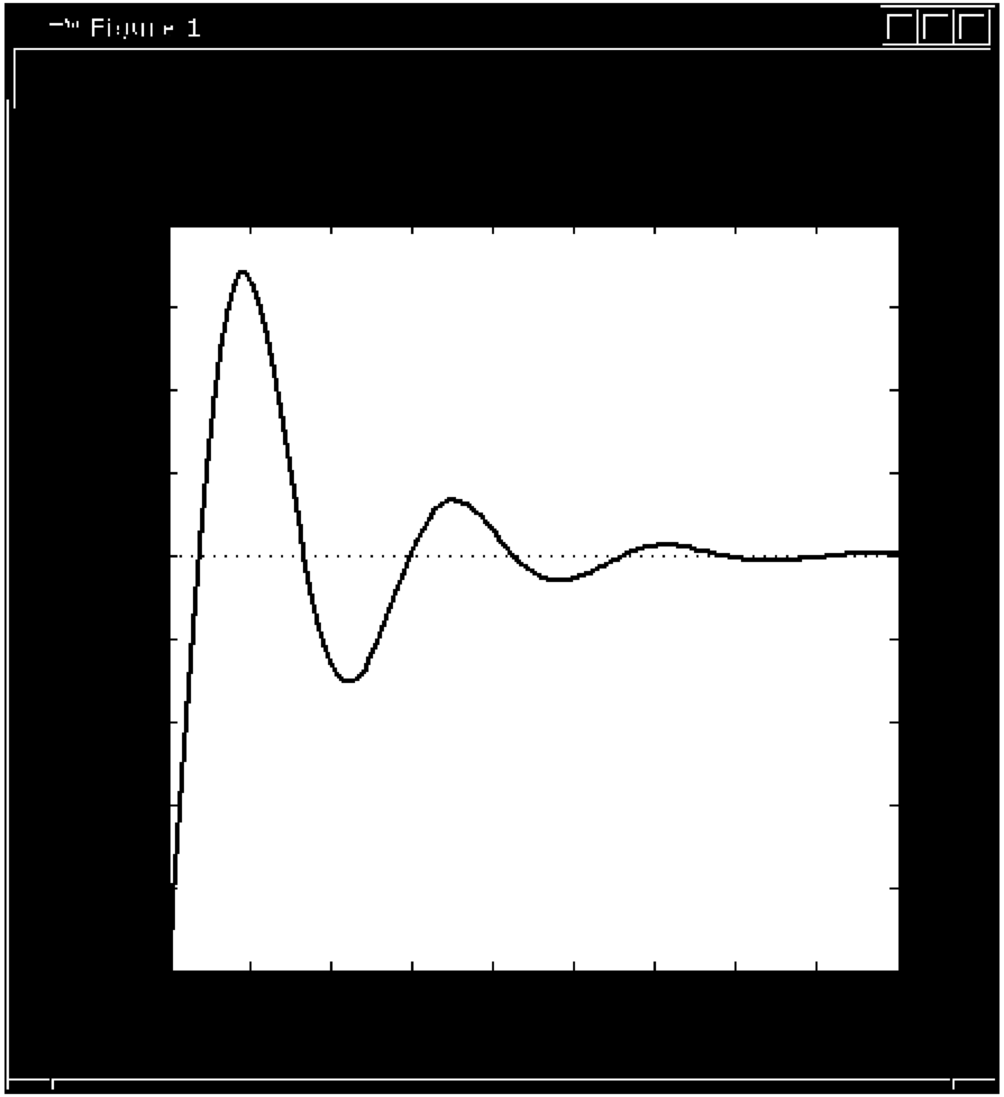
$$Backward = V \tag{8}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{\frac{1}{sT_i}}{1 + V\frac{1}{sT_i}} \equiv$$

$$\equiv \texttt{feedback(tf(1,[Ti 0]), tf(V, 1))} \tag{9}$$

Matlab Functions – Step Response

```
system=tf([2 1],[4 1 1]);
t=0:0.1:50;
step(100*system)
axis([0 30 60 180])
```



Steady-State Error – Definition

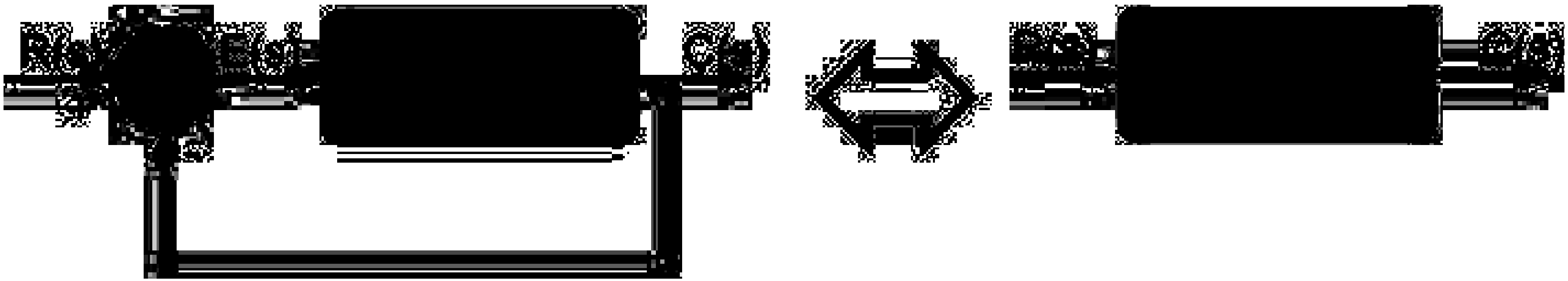


Figure 1: Unity Feedback System

Steady-State Error

The difference between the input and output of a system in the limit as time goes to infinity

Steady-State Error

Steady-State Error

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)} \tag{10}$$

$$e(\infty) = \lim_{s \to 0} sR(s) \left|1 - T(s)\right| \tag{11}$$

Feedback controller – How does it work I?

Figure 2: System controller

- e – represents the tracking error
- e – difference between desired input (R) an actual output (Y)
- e – is sent to controller which computes:
  - derivative of e
  - integral of e
- u – controller output is equal to...

Feedback controller – How does it work II?

- u – controller output is equal to:
  - K_p (proportional gain) times the magnitude of the error +
  - K_i (integral gain) times the integral of the error +
  - K_d (derivative gain) times the derivative of the error

Controller's Output

$$u = K_p e + K_i \int e dt + K_d \frac{de}{dt}$$

Controller's Transfer Function

$$K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$$

Characteristics of PID Controllers

- Proportional Controller K_p
  - reduces the rise time
  - reduces but never eliminates steady-state error
- Integral Controller K_i
  - eliminates steady-state error
  - worsens transient response
- Derivative Controller K_d
  - increases the stability of the system
  - reduces overshoot
  - improves transient response

Example Problem

Figure 3: Mass spring and damper problem

Modelling Equation

$$m\ddot{x} + b\dot{x} + kx = F \tag{12}$$

Example Problem

Laplace & Transfer Functions

$$m\ddot{x} + b\dot{x} + kx = F$$

$$ms^2X(s) + bsX(s) + kX(s) = F(s) \tag{13}$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \tag{14}$$

MATLAB System Response

Assumptions

Let: $m = 1[kg]$, $b = 10[Ns/m]$, $k = 20[N/m]$

MATLAB code

```
%{Set up variables%}
m=1; b=10; k=20;
%{Calculate response%}
num=1;
den=[m, b, k];
plant=tf(num,den);
step(plant)
```

MATLAB System Response

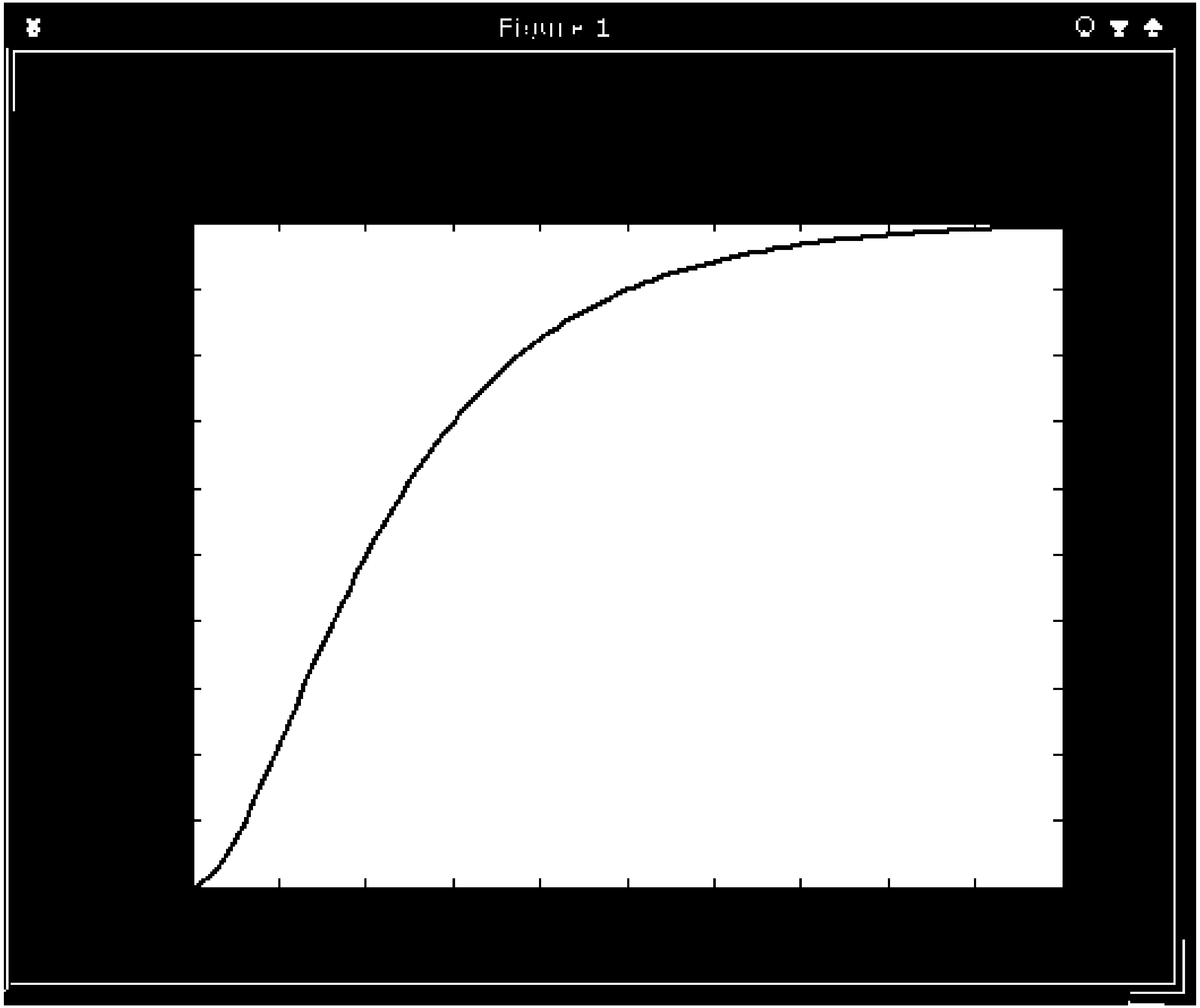


Figure 4: Amplitude ⇔ Displacement

Problems

- The steady-state error is equal to 0.95 – equation (11)
- The rise time is about 1 second
- The settling time is about 1.5 seconds
- The PID controller should influence (reduce) all those parameters

Controllers' Characteristics

Type	Rise time	Overshoot	Settling time	S-S Error
K_p	decrease	increase	small change	decrease
K_i	decrease	increase	increase	eliminate
K_d	small change	decrease	decrease	small change

These correlations may not be exactly accurate, because K_p, K_i, and K_d are dependent on each other. In fact, changing one of these variables can change the effect of the other two.

Proportional Controller

P Transfer Function

$$\frac{X(s)}{F(s)} = \frac{K_p}{s^2 + bs + (k + K_p)}$$

Matlab code

```
%{Set up proportional gain%}
Kp=300;
%{Calculate controller%}
sys_ctl=feedback(Kp*plant, 1);
%{Plot results%}
t=0:0.01:2;
step(sys_ctl, t)
```

Proportional Controller – Plot

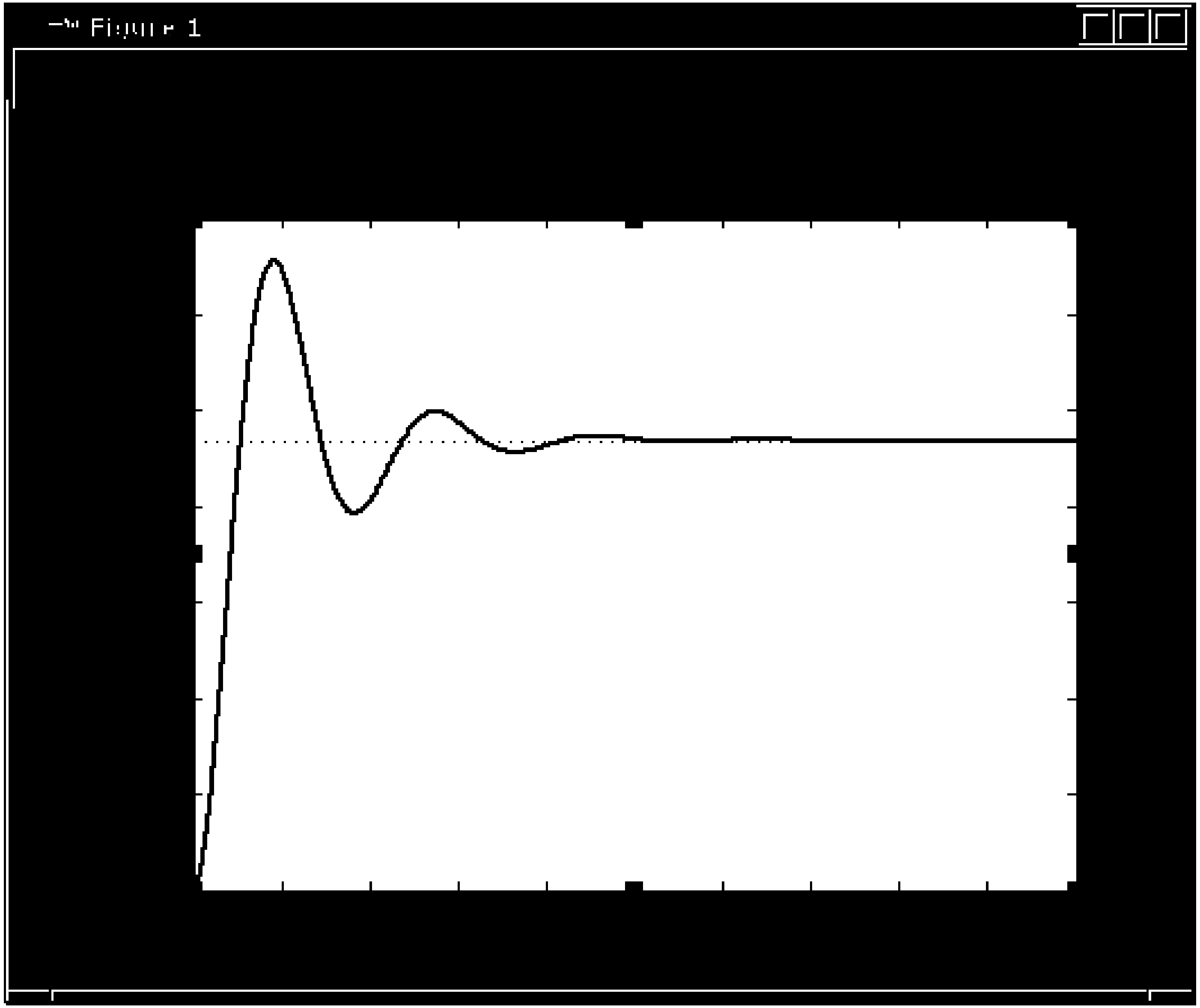


Figure 5: Improved rise time & steady-state error

Proportional Derivative Controller

PD Transfer Function

$$\frac{X(s)}{F(s)} = \frac{K_d s + K_p}{s^2 + (b + K_d)s + (k + K_p)}$$

MATLAB code

```
%{Set up proportional and derivative gain%}
Kp=300;  Kd=10;
%{Calculate controller%}
contr=tf([Kd, Kp],1);
sys_ctl=feedback(contr*plant, 1);
%{Plot results%}
t=0:0.01:2;
step(sys_ctl, t)
```

Proportional Derivative Controller – Plot

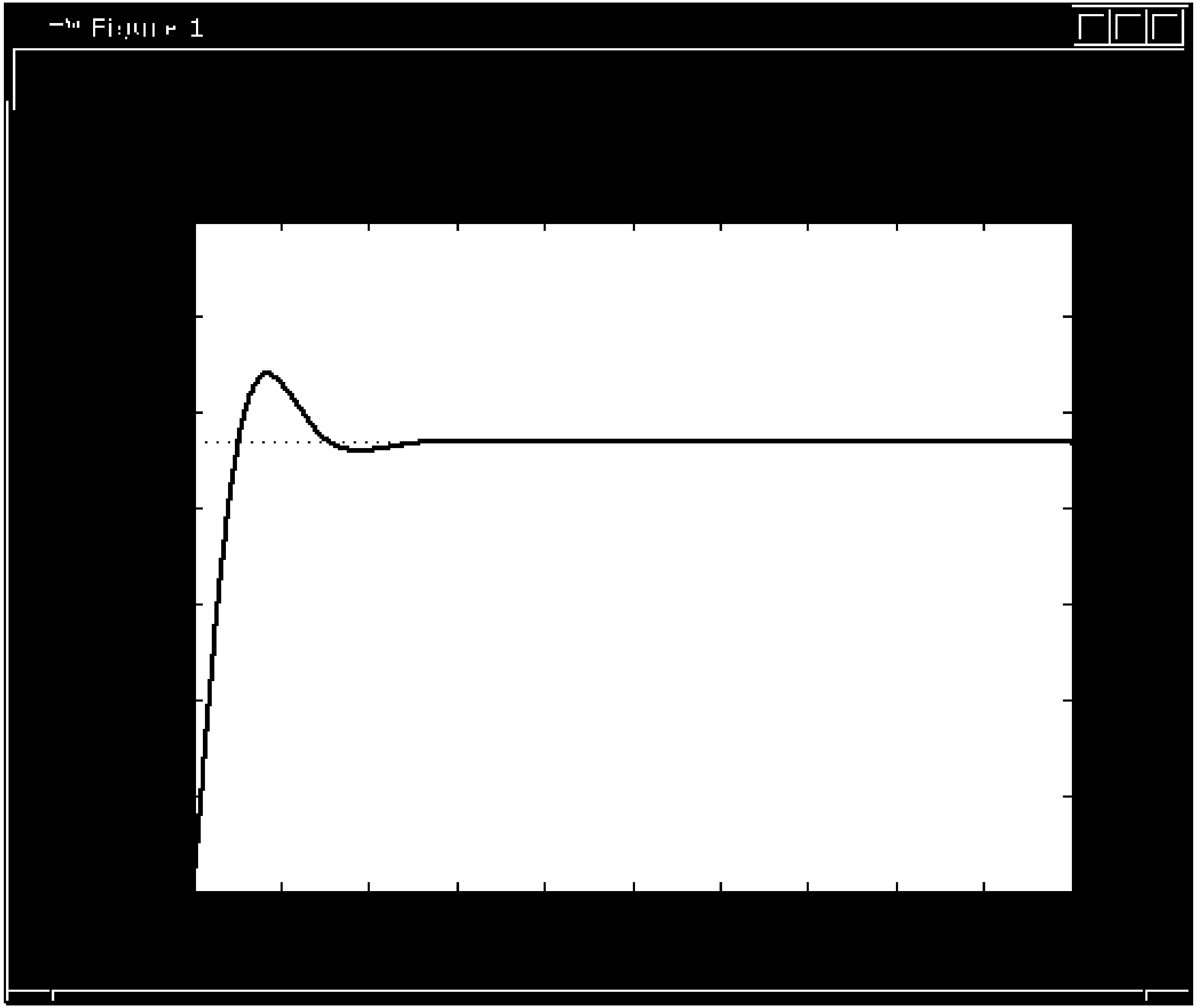


Figure 6: Reduced over-shoot and settling time

Proportional Integral Controller

PI Transfer Function

$$\frac{X(s)}{F(s)} = \frac{K_p s + K_i}{s^3 + bs^2 + (k + K_p)s + K_i}$$

MATLAB code

```
%{Set up proportional and integral gain%}
Kp=30;    Ki=70;
%{Calculate controller%}
contr=tf([Kp, Ki],[1, 0]);
sys_ctl=feedback(contr*plant, 1);
%{Plot results%}
t=0:0.01:2;
step(sys_ctl, t)
```

Proportional Integral Controller – Plot

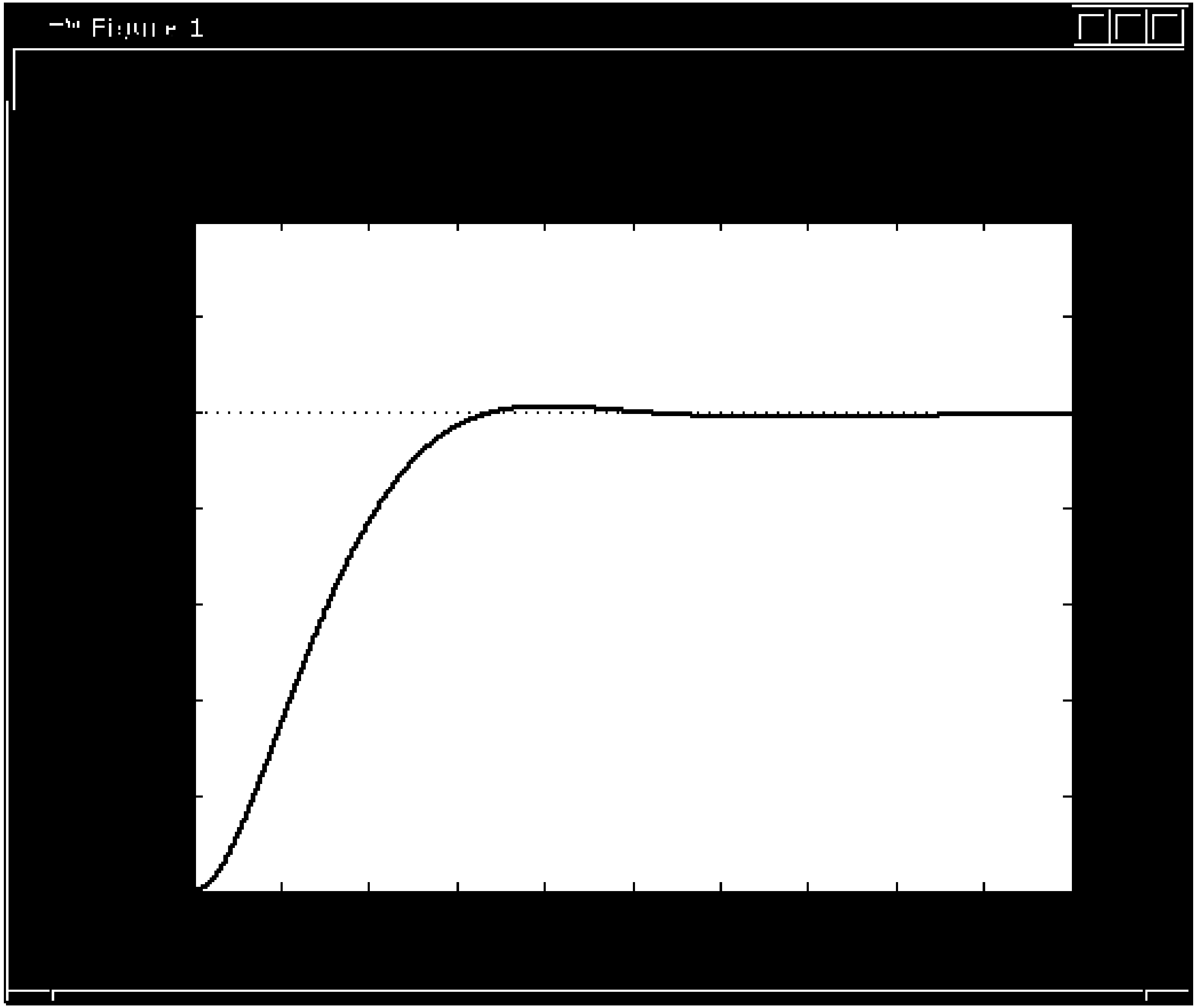


Figure 7: Eliminated steady-state error, decreased over-shoot

Proportional Integral Derivative Controller

PID Transfer Function

$$\frac{X(s)}{F(s)} = \frac{K_d s^2 + K_p s + K_i}{s^3 + (b + K_d)s^2 + (k + K_p)s + K_i}$$

MATLAB code

```
%{Set up proportional and integral gain%}
Kp=350;  Ki=300; Kd=50;
%{Calculate controller%}
contr=tf([Kd, Kp, Ki],[1, 0]);
sys_ctl=feedback(contr*plant, 1);
%{Plot results%}
t=0:0.01:2;
step(sys_ctl, t)
```

Proportional Integral Derivative Controller – Plot

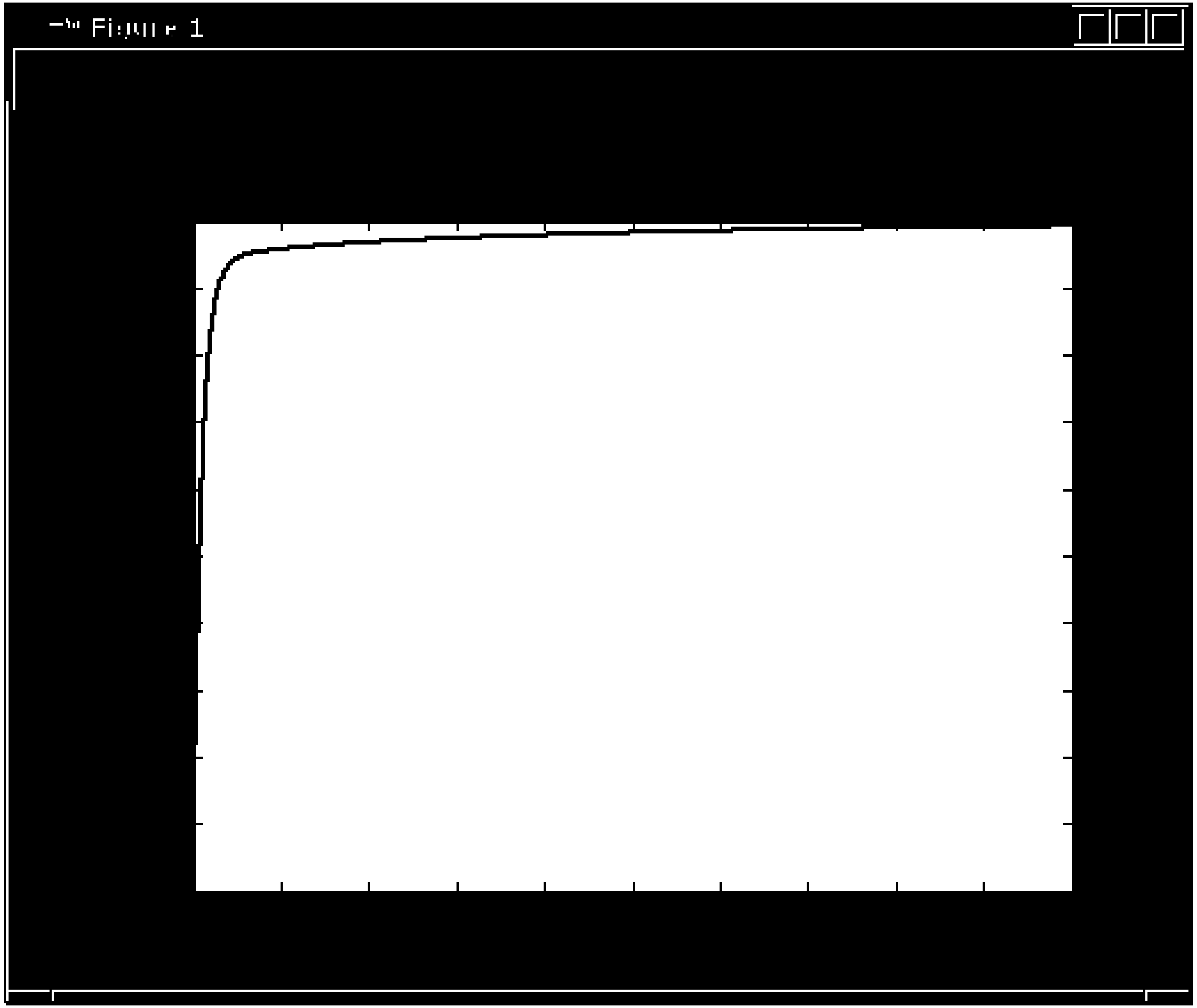


Figure 8: Eliminated steady-state error, decreased over-shoot

Part III

Matlab – Cruise Control System

How does Cruise Control for Poor work?

Figure 9: Forces taking part in car's movement

Based on Carnegie Mellon University Library Control Tutorials for MATLAB and Simulink

Building the Model

Using Newton's law we derive

$$F = m\dot{v} + bv \tag{15}$$

$$y = v \tag{16}$$

Where: $m = 1200[kg]$, $b = 50[\frac{Ns}{m}]$, $F = 500[N]$

Design Criteria

- For the given data $V_{max} = 10[m/s] = 36[km/h]$
- The car should accelerate to V_{max} within $6[s]$
- 10% tolerance on the initial velocity
- 2% of a steady-state error

Transfer Function

System Equations:

$$F = m\dot{v} + bv$$
$$y = v$$

Laplace Transform:

$$F(s) = msV(s) + bV(s) \tag{17}$$
$$Y(s) = V(s) \tag{18}$$

Transfer Function:

$$\frac{Y(s)}{F(s)} = \frac{1}{ms + b} \tag{19}$$

MATLAB Representation

- Now in MATLAB we need to type

MATLAB code

```
m=1200;
b=50;
num=[1];
den=[m,b];
cruise=tf(num, den);
step = (500*cruise);
```

Results

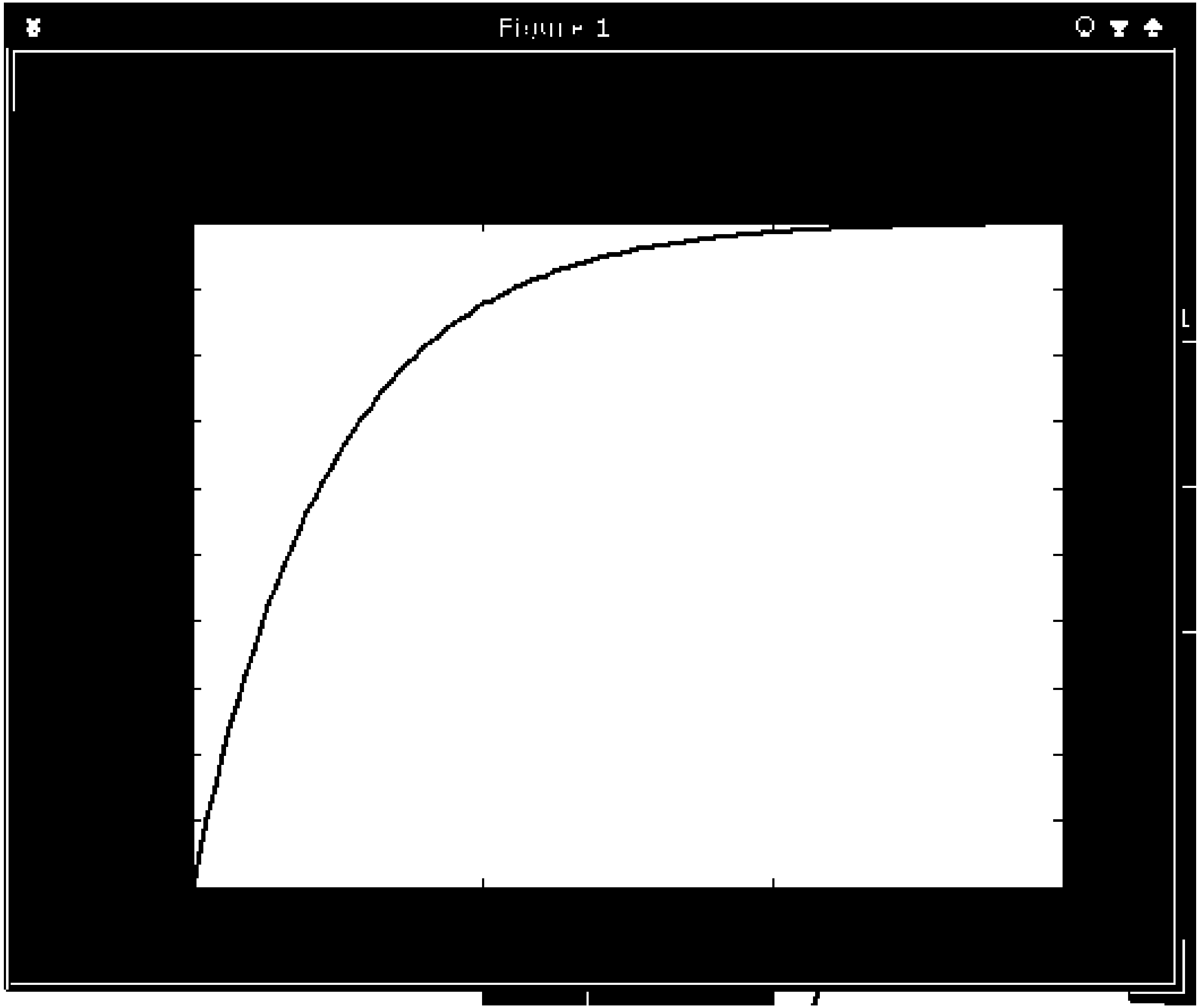


Figure 10: Car velocity diagram – mind the design criteria

Design criteria revisited

- Our model needs over 100[s] to reach the steady-state
- The design criteria mentioned 5 seconds

Feedback controller

- To adjust the car speed within the limits of specification
- We need the feedback controller

Figure 11: System controller

Decreasing the rise time

Proportional Controller

$$\frac{Y(s)}{R(s)} = \frac{K_p}{ms + (b + K_p)} \tag{20}$$

MATLAB code

```
Kp=100; m=1200; b=50;
num=[1]; den=[m,b];
cruise=tf(num, den);
sys_ctl=feedback(Kp*cruise , 1);
t=0:0.1:20;
step(10*sys_cl ,t)
axis([0 20 0 10])
```

Under- and Overcontrol

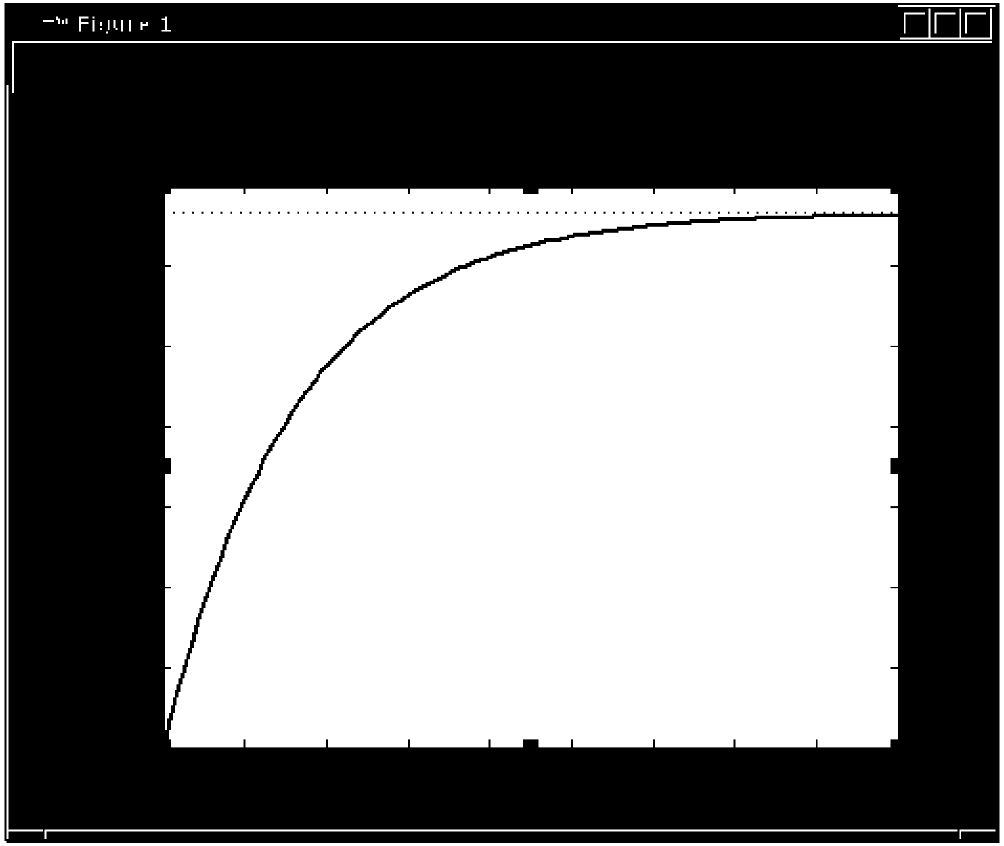


Figure 12: $K_p = 100$

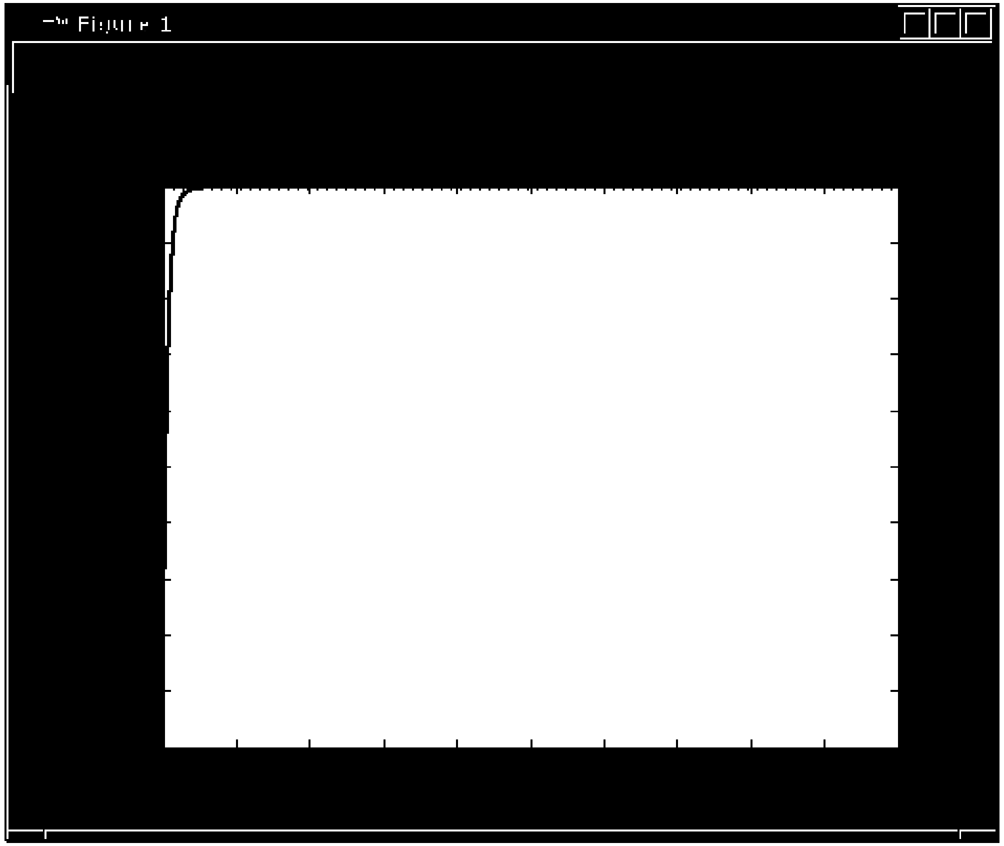


Figure 13: $K_p = 10000$

Making Rise Time Reasonable

Proportional Integral Controller

$$\frac{Y(s)}{R(s)} = \frac{K_p s + K_i}{ms^2 + (b + K_p)s + K_i} \tag{21}$$

Matlab code

```
Kp=800; Ki=40; m=1200; b=50;
num=[1]; den=[m,b];
cruise=tf(num, den);
contr=tf([Kp Ki],[1 0])
sys_ctl=feedback(contr*cruise, 1);
t=0:0.1:20;
step(10*sys_cl,t)
axis([0 20 0 10])
```

Results

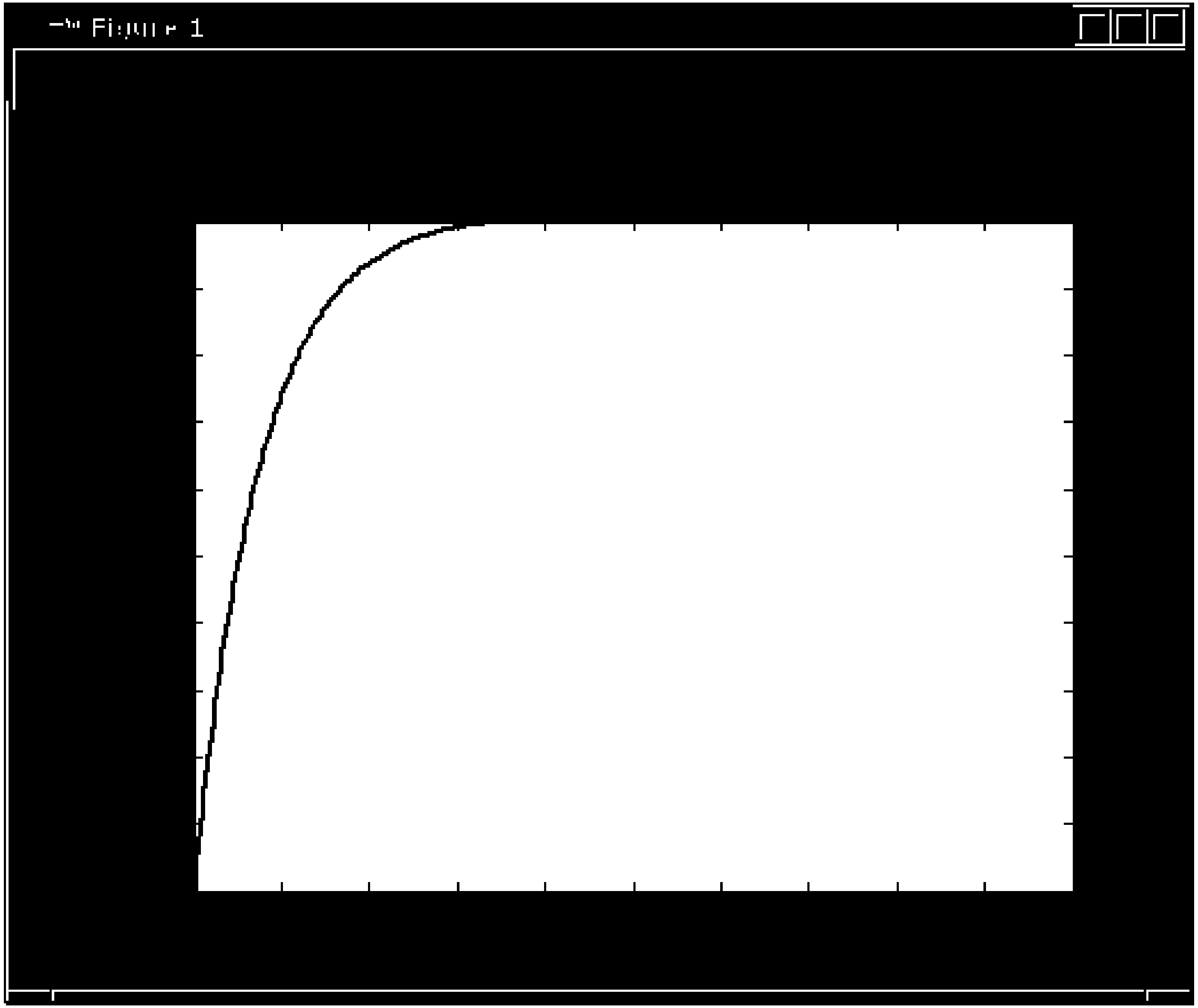


Figure 14: Car velocity diagram meeting the design criteria

Part IV

Simulink – Cruise Control System

How does Cruise Control for Poor work?

Figure 15: Forces taking part in car's movement

Based on Carnegie Mellon University Library Control Tutorials for MATLAB and Simulink

Physical Description

- Summing up all the forces acting on the mass

Forces acting on the mass

$$F = m\frac{dv}{dt} + bv \tag{22}$$

Where: m=1200[kg], b=50[$\frac{Nsec}{m}$], F=500[N]

Physical Description – cntd.

- Integrating the acceleration to obtain the velocity

Integral of acceleration

$$a = \frac{dv}{dt} \equiv \int \frac{dv}{dt} = v \tag{23}$$

Building the Model in Simulink

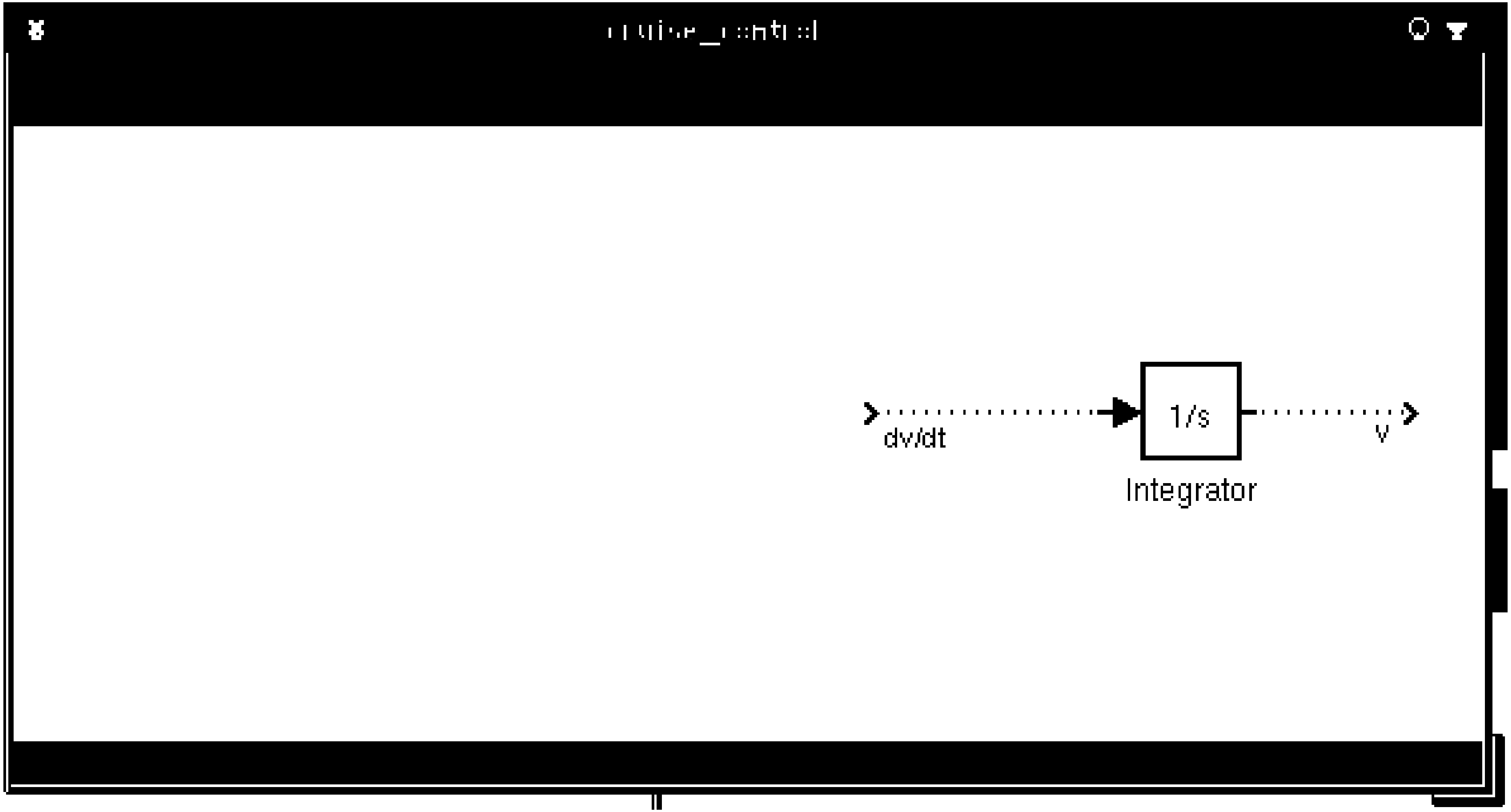


Figure 16: Integrator block from Continuous block library

Building the Model in Simulink

- Obtaining acceleration

Acceleration

$$a = \frac{dv}{dt} = \frac{F - bv}{m} \tag{24}$$

Building the Model in Simulink

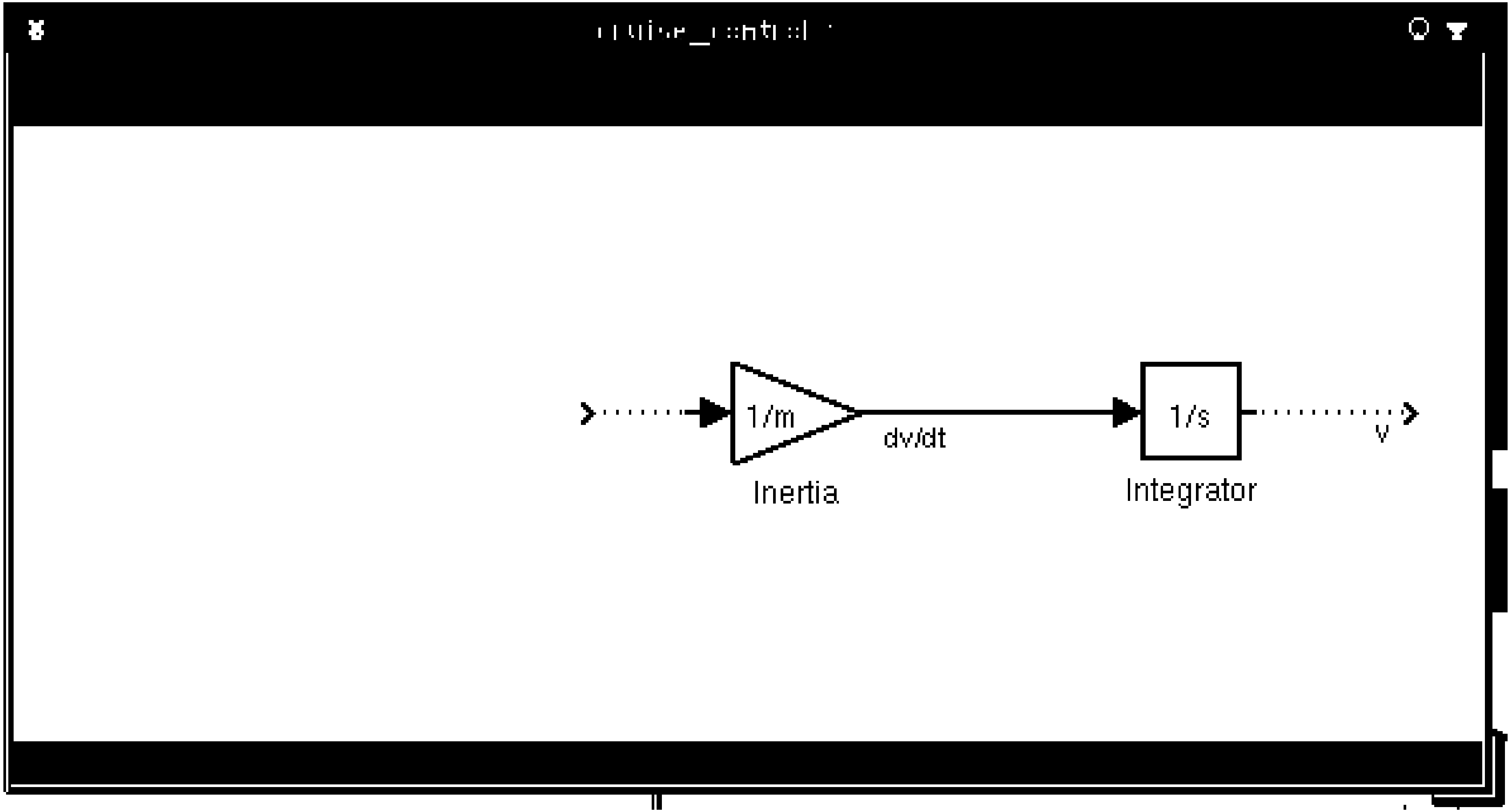
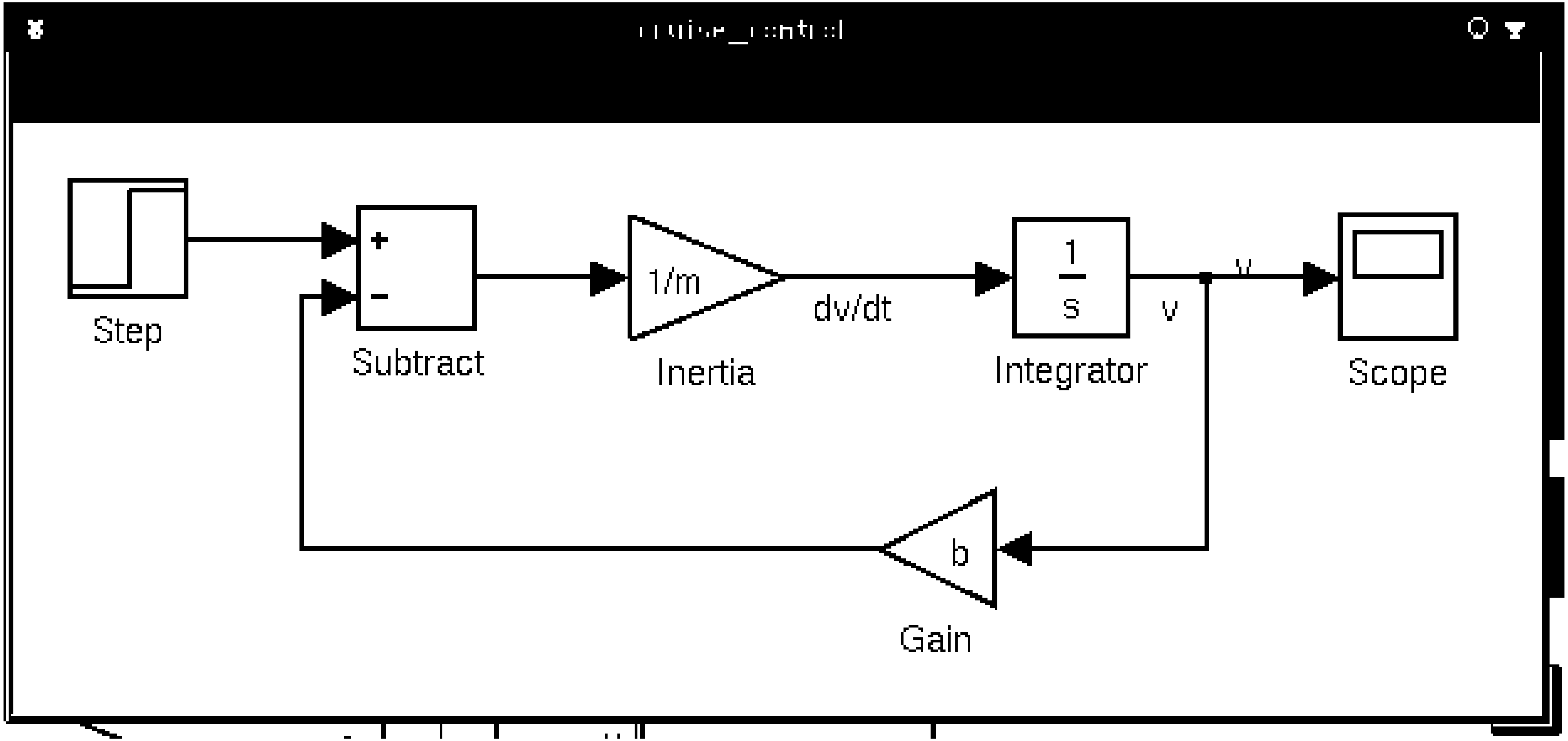


Figure 17: Gain block from Math operators block library

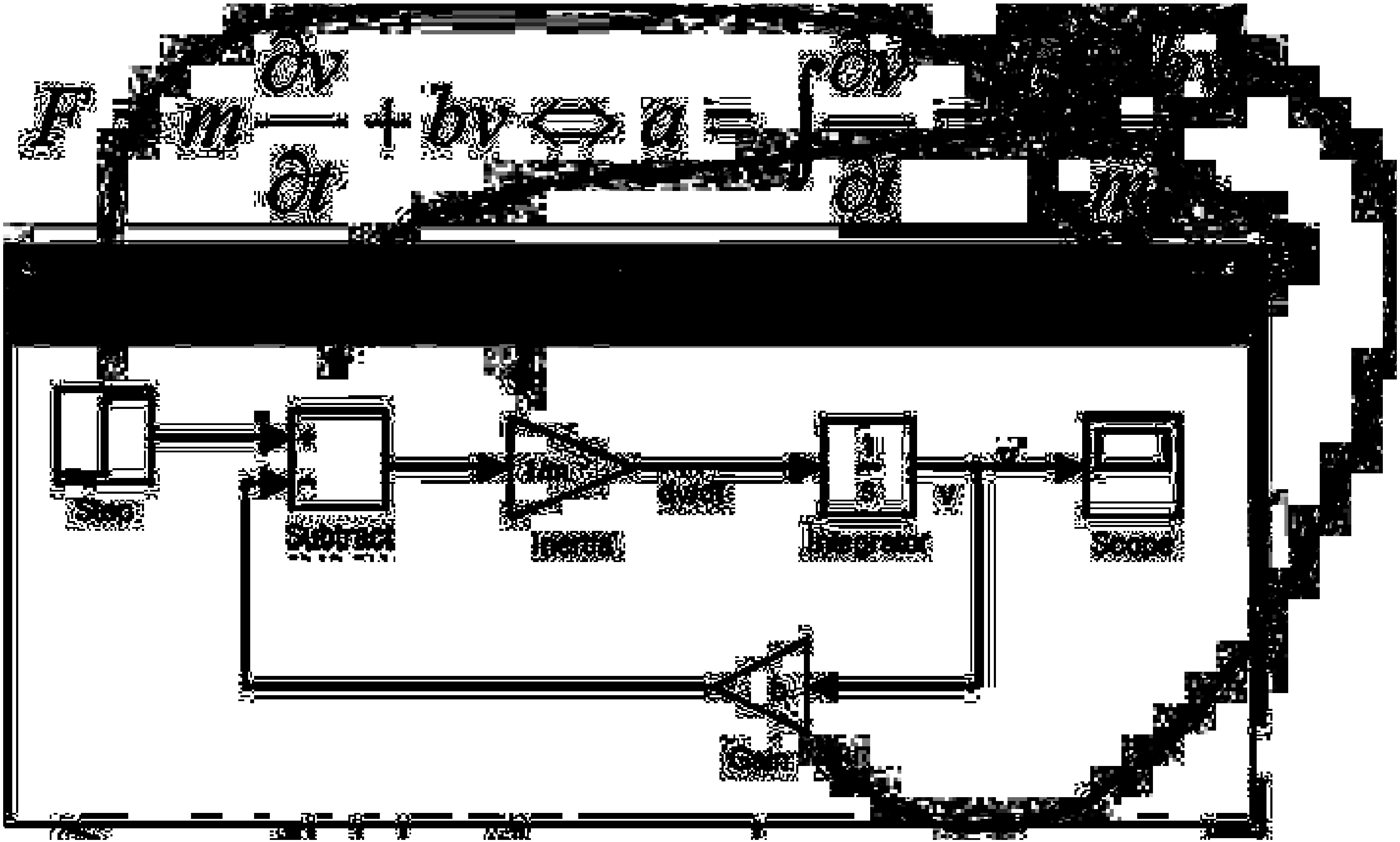
Elements used in Simulink Model

- Friction (Gain block)
- Subtract (from Math Operators)
- Input (Step block from Sources)
- Output (Scope from Sinks)

Complete Model



Mapping Physical Equation to Simulink Model



Setting up the Variables

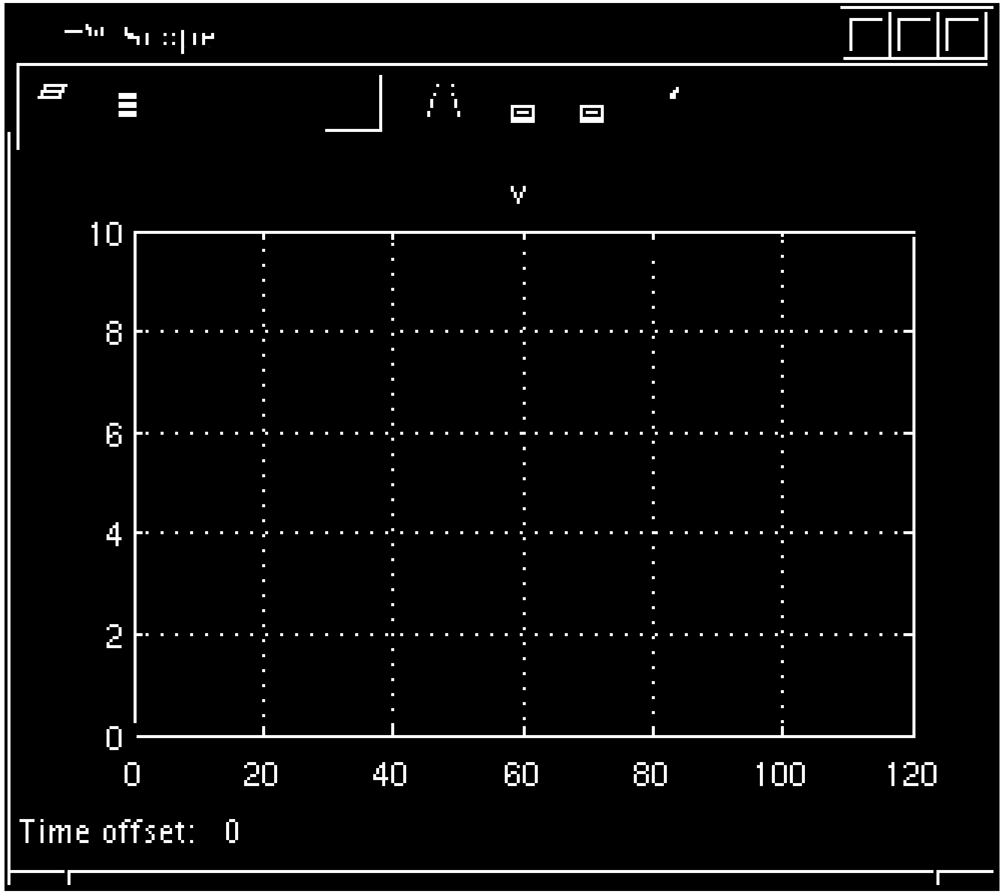
- Now it is time to use our input values in Simulink...
  - F=500[N]
  - In Step block set: Step time = 0 and Final value = 500
- ...and adjust simulation parameters...
  - Simulation → Configuration Parameters...
  - Stop time = 120

...and set up variables in MATLAB

```
m=1200;
b=50;
```

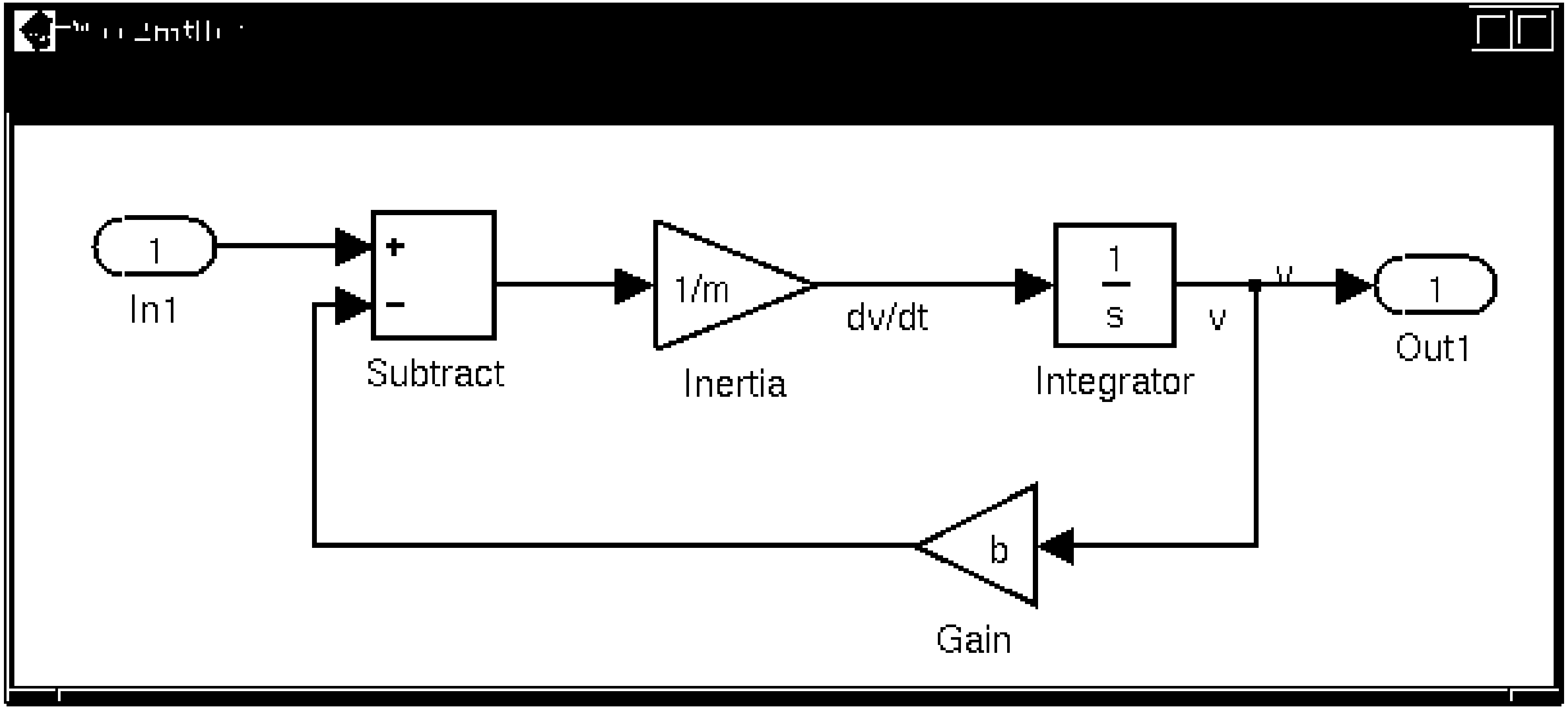
Running Simulation

- Choose Simulation→Start
- Double-click on the Scope block...



Extracting Model into MATLAB

- Replace the Step and Scope Blocks with In and Out Connection Blocks



Verifying Extracted Model

- We can convert extracted model
  - into set of linear equations
  - into transfer function

MATLAB code

```
[A, B, C, D]=linmod('cc2mtlb');
[num, den]=ss2tf(A, B, C, D);
step(500*tf(num, den));
```

MATLAB vs Simulink

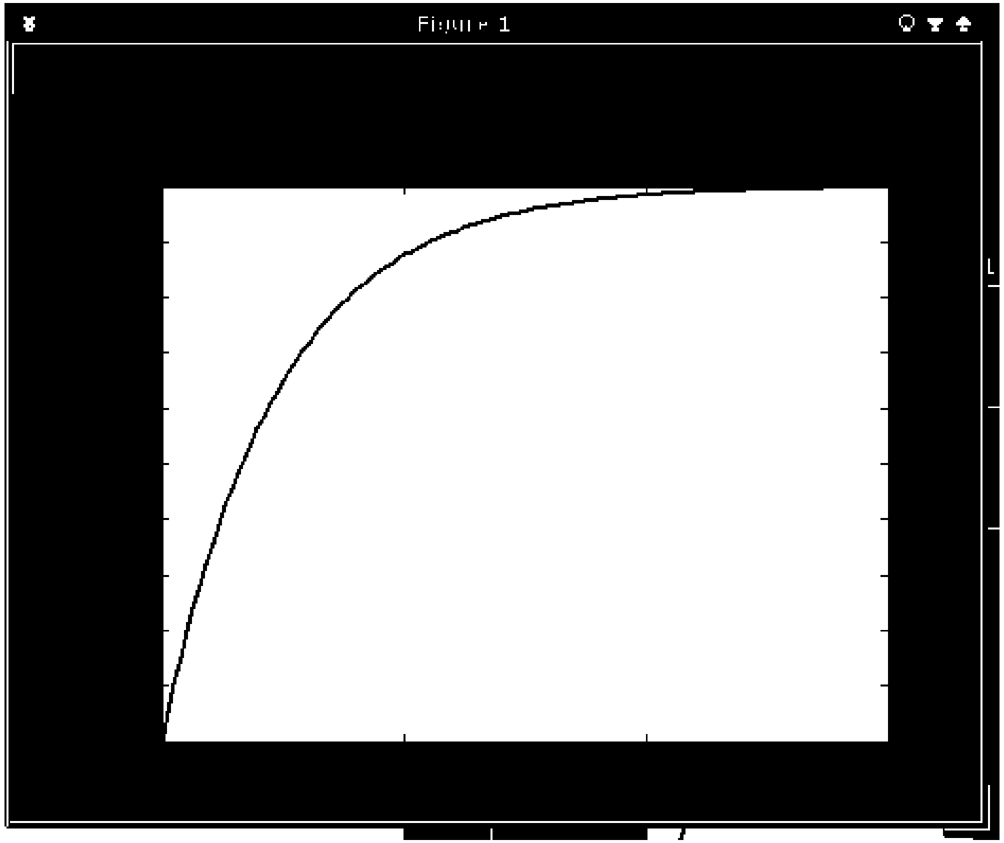


Figure 18: MATLAB

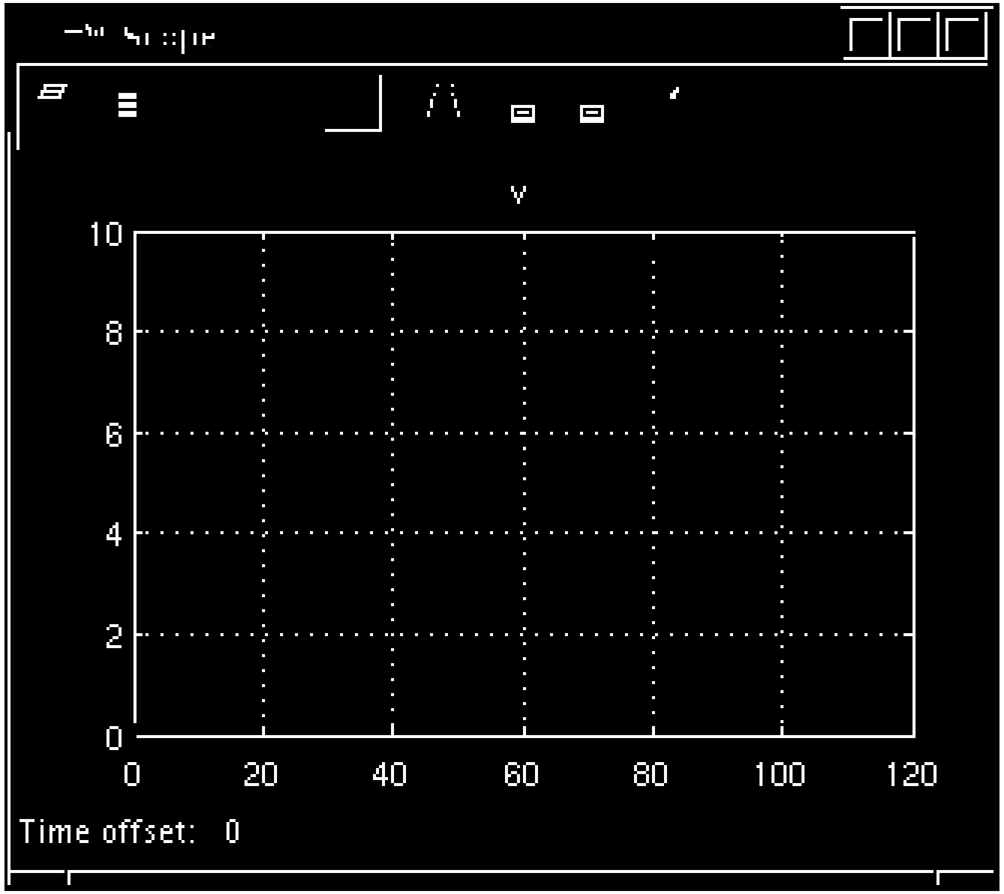


Figure 19: Simulink

The open-loop system

- In MATLAB section we have designed a PI Controller
  - $K_p = 800$
  - $K_i = 40$
- We will do the same in Simulik
- First we need to contain our previous system in a Sybsystem block
- Choose a Subsystem block from the Ports&Subsystems Library
- Copy in the model we used with MATLAB

Subsystem

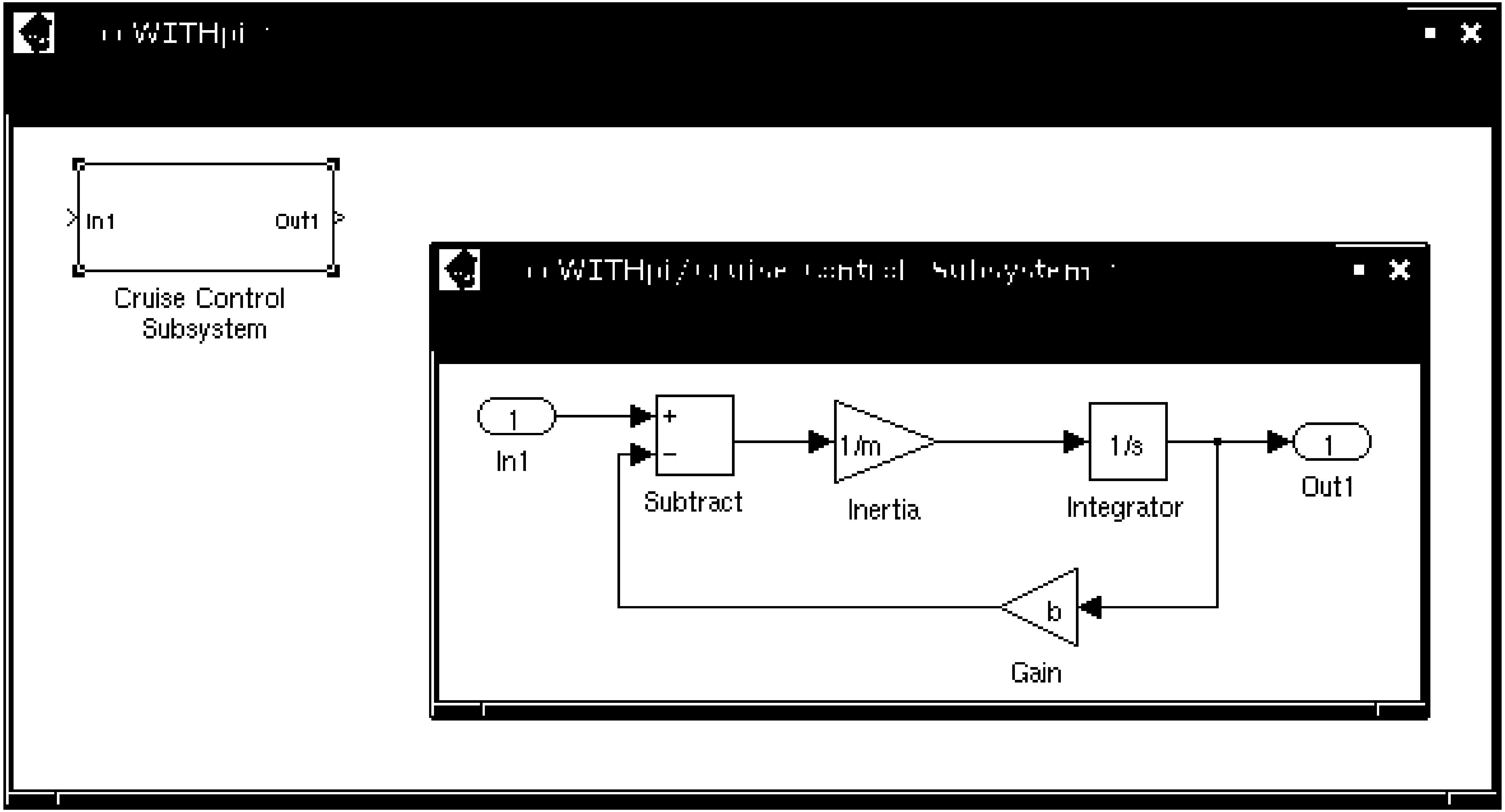


Figure 20: Subsystem Block and its Contents

PI Controller I

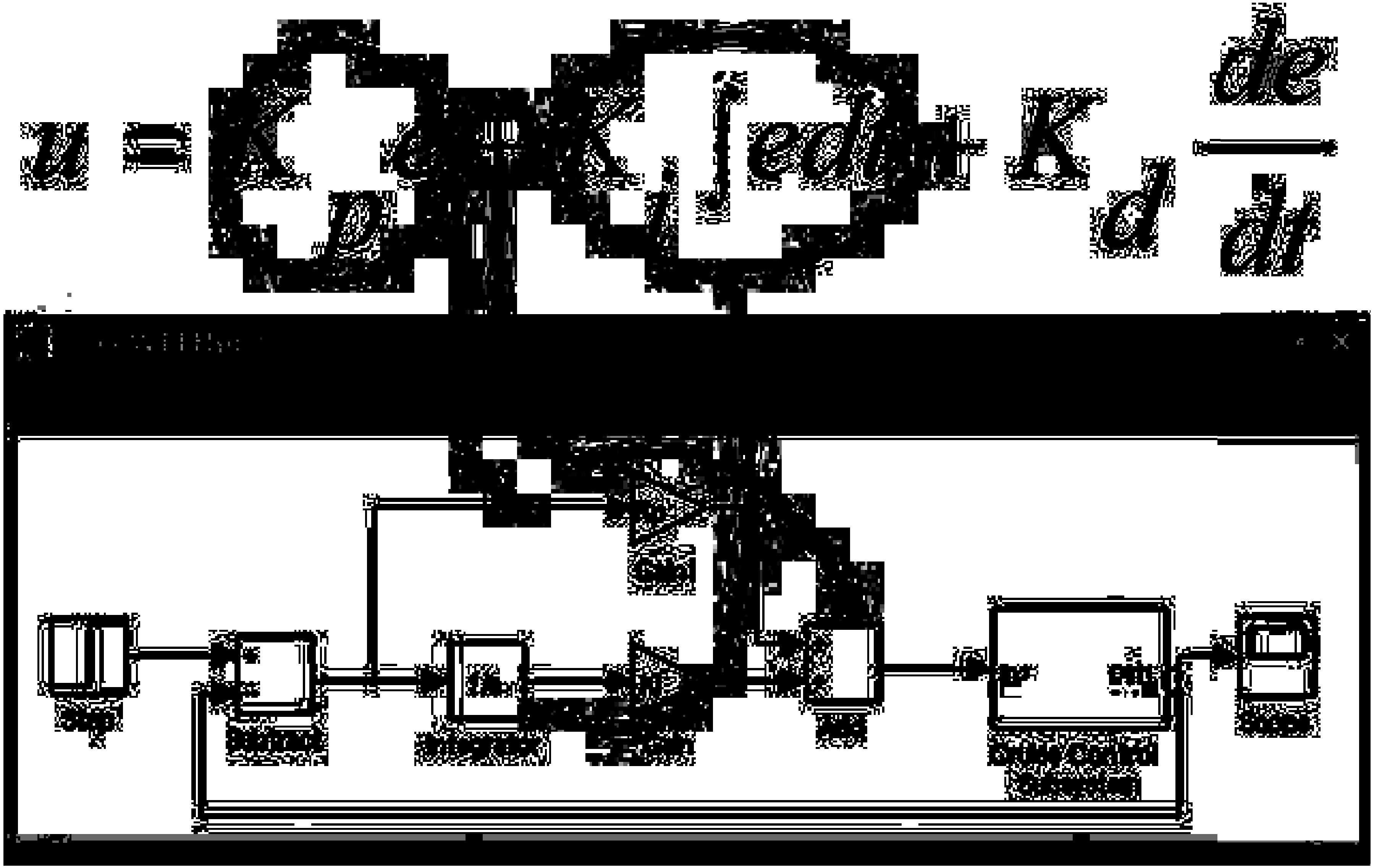


Figure 21: Step: final value=10, time=0

PI Controller II

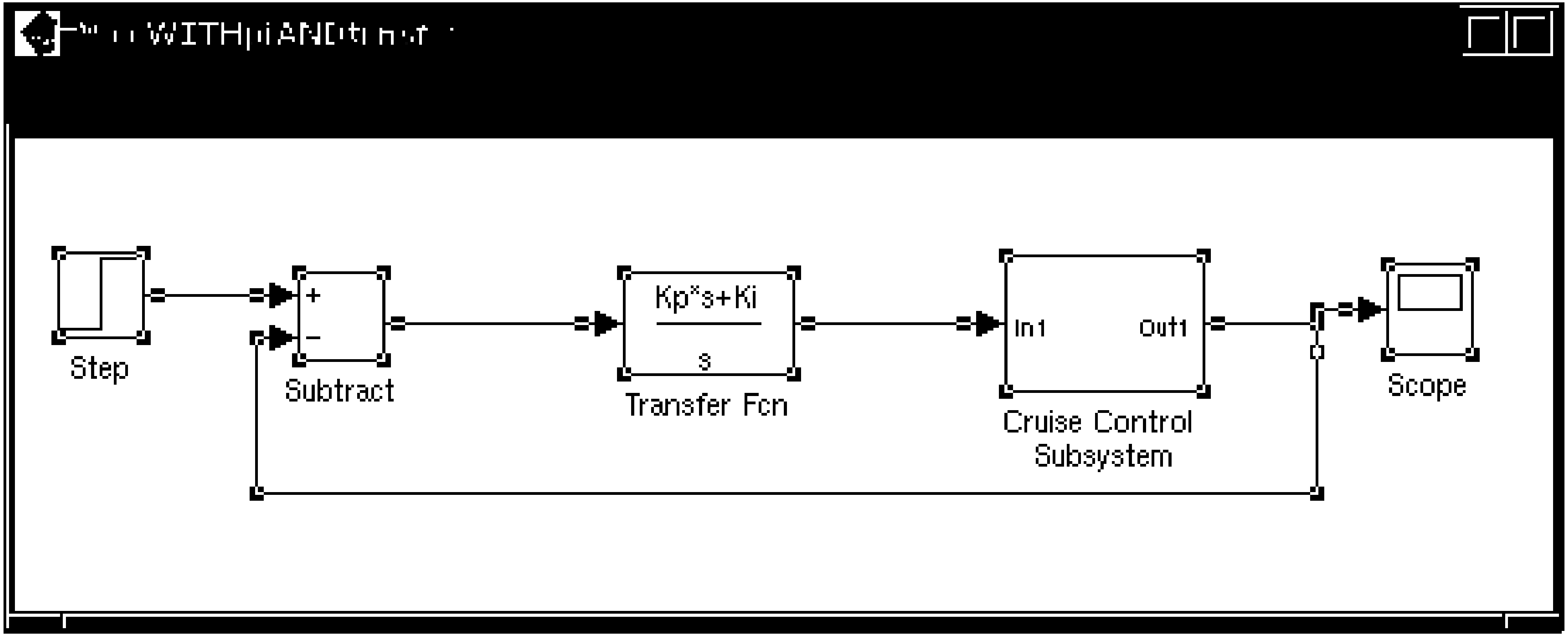
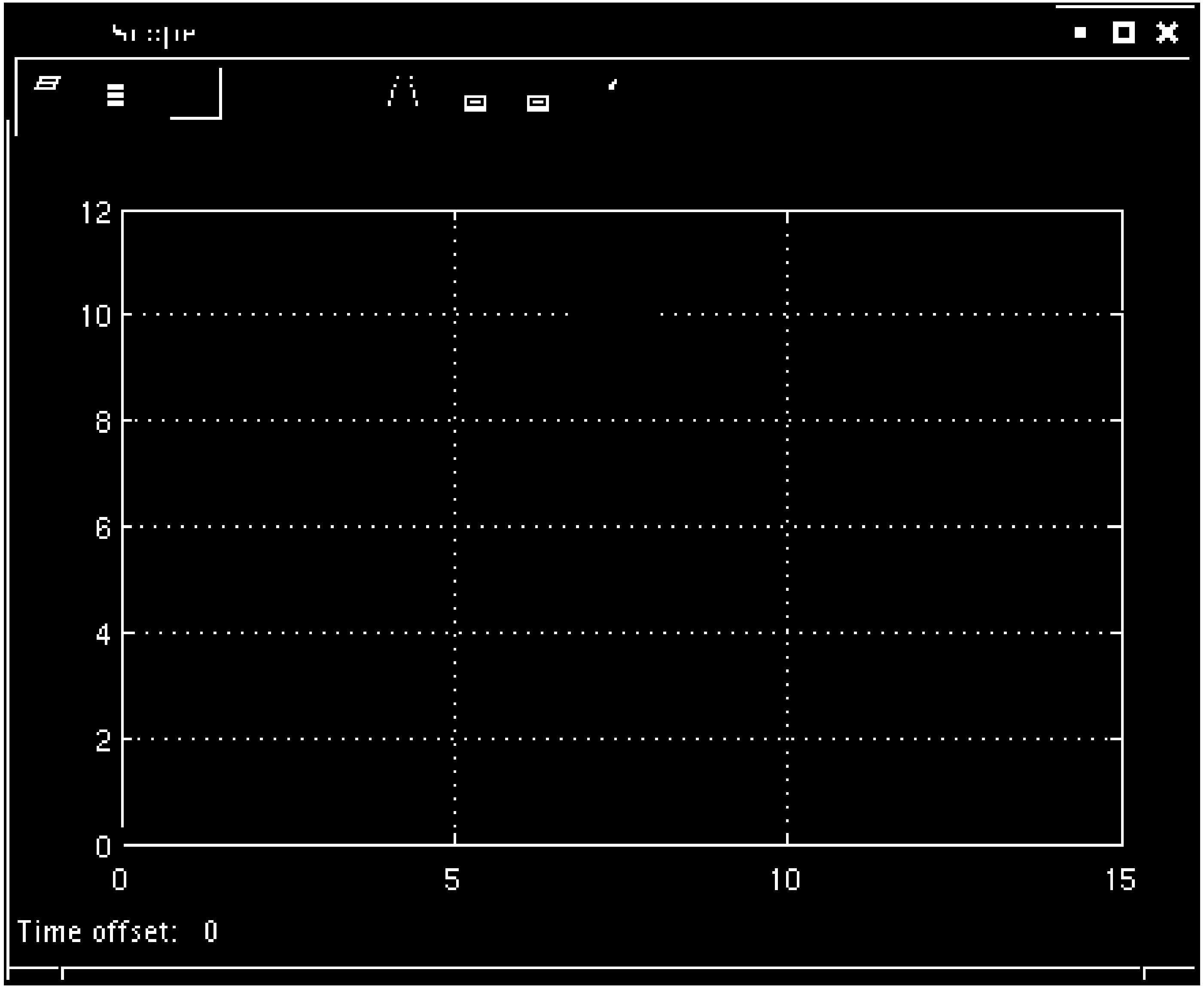


Figure 22: We use Transfer Fcn block from Continuous-Time Linear Systems Library

Results

- Runnig simulation with time set to 15[s]



References

Course basic references

Textbooks

- *Digital Control of Dynamic Systems* (3rd Edition) by Gene F. Franklin, J. David Powell, Michael L. Workman Publisher: Prentice Hall; 3 edition (December 29, 1997) ISBN: 0201820544

- Lecture slides
- Computer Lab Exercises