ID6

ARMAX Identification

6.18 MULTIVARIABLE ARMAX PREDICTORS

The MFD ARMAX model (6.17.8) is constituted, like the MFD ARX model (3.17.3), by m non synchronous relations of the type

$$\begin{aligned} y_i(t+\nu_i) = & \sum_{j=1}^{m}\sum_{k=1}^{\nu_{ij}} \alpha_{ijk}\big(y_j(t+k-1) - w_j(t+k-1)\big) \\ & + \sum_{j=1}^{r}\sum_{k=1}^{\nu_i} \beta_{ijk}\, u_j(t+k-1) \\ & + \sum_{j=1}^{m}\sum_{k=1}^{\nu_i} \gamma_{ijk}\, w_j(t+k-1) + w_i(t+\nu_i) \quad (i=1,\dots,m) \end{aligned} \tag{6.18.1}$$

that prevent the direct construction of a predictor. Also in this case it is possible to construct, as has been done in ID3.17, an optimal predictor preserving the minimal parameterization of (6.17.8) by premultiplying equation (6.17.3) by the non singular and non unimodular matrix $M(z)$ (3.17.5). We obtain the new equivalent (but not strictly equivalent) MFD ARMAX model

$$Q(z)^* y(t) = P(z)^* u(t) + \big(Q(z)^* + R(z)^*\big) w(t) = P(z)^* u(t) + S(z)^* w(t). \tag{6.18.2}$$

where

$$Q(z)^* = M(z)\, Q(z) = Q^*_{\nu_M+1}\, z^{\nu_M} - Q^*_{\nu_M}\, z^{\nu_M-1} - \dots - Q^*_1 \tag{6.18.3a}$$

$$P(z)^* = M(z)\, P(z) = P^*_{\nu_M}\, z^{\nu_M-1} + \dots + P^*_2 z + P^*_1 \tag{6.18.3b}$$

$$R(z)^* = M(z)\, R(z) = R^*_{\nu_M}\, z^{\nu_M-1} + \dots + R^*_2 z + R^*_1. \tag{6.18.3c}$$

Imposing a prevision error equal to $w(t)$,

$$y(t) - y(t|t-1) = w(t), \tag{3.18.4}$$

we obtain the expression of the optimal predictor

$$\left[Q^*(z) + R^*(z)\right] y(t|t-1) = R^*(z)\, y(t) + P^*(z)\, u(t) \tag{3.18.5}$$

or, equivalently,

$$\begin{aligned} y(t|t-1) = Q^{*-1}_{\nu_M+1} & \sum_{i=1}^{\nu_M} R^*_{\nu_M+1-i}\, y(t-i) + \sum_{i=1}^{\nu_M} P^*_{\nu_M+1-i}\, u(t-i) \\ & + \sum_{i=1}^{\nu_M} \left(Q^*_{\nu_M+1-i} - R^*_{\nu_M+1-i}\right) y(t-i|t-i-1)\,. \end{aligned} \tag{3.18.6}$$

See Remark 3.17.1 to recall the properties of $Q^*_{\nu_M+1}$.