ID6

ARMAX Identification

6.2 ARMAX PREDICTORS

The expression of the optimal predictor for an ARMAX process can be deduced observing that the only term in relation (6.1.2) that cannot be predicted at time $t-1$ is $w(t)$, because of the whiteness of $w(\cdot)$. Imposing $w(t)$ as prediction error, i.e.

$$y(t) - y(t|t-1) = w(t) \tag{6.2.1}$$

and using (6.2.1) to substitute the terms $w(t), w(t-1), \ldots$ in (6.1.2), we obtain

$$\begin{aligned} y(t|t-1) = {} & (\alpha_n + \gamma_n)\, y(t-1) + \ldots + (\alpha_1 + \gamma_1)\, y(t-n) + \beta_n u(t-1) + \ldots \\ & + \beta_1 u(t-n) - \gamma_n\, y(t-1|t-2) - \ldots - \gamma_1\, y(t-n|t-n-1). \end{aligned} \tag{6.2.2}$$

The expression of the optimal one–step–ahead predictor (6.2.2) can also be written, using forward notations, as

$$r(z)\, y(t|t-1) = \big(r(z) - q(z)\big)\, y(t) + p(z)\, u(t) \tag{6.2.3}$$

$$y(t|t-1) = \frac{\big(r(z) - q(z)\big)}{r(z)}\, y(t) + \frac{p(z)}{r(z)}\, u(t); \tag{6.2.4}$$

similar expressions can be obtained using a backward notation. It can be noted that, differently from ARX predictors, optimal ARMAX predictor (6.2.2) makes use of previous predictions and is thus subject to stability conditions: all zeros of $r(z)$ must lie inside the unit circle. This condition is always met by MA processes with rational spectra.