LA

Linear Algebra

LA.7 POLYNOMIAL MATRICES

Definition LA.7.1 (Polynomial matrices) – A polynomial matrix $M(z)$ is a matrix whose entries belong to the commutative ring of polynomials in z of finite degree with coefficients in $\mathcal{R}$.

Property LA.7.1 – The inverse of a polynomial matrix $M(z)$, given by

$$M(z)^{-1} = \frac{\text{agg}\, M(z)}{\det M(z)}, \qquad \text{(LA.7.1)}$$

is, in general, a rational matrix.

Definition LA.7.2 (Elementary row and column operations) – The following elementary row and column operations are defined on polynomial matrices:

- Interchange of two rows (columns);
- Multiplication of a row (column) by a nonzero scalar in $\mathcal{R}$;
- Substitution of a row (column) with its sum with another row (column) multiplied by any polynomial.

Definition LA.7.3 (Unimodular matrices) – A unimodular matrix $U(z)$ is defined as any square polynomial matrix whose determinant is a nonzero scalar in $\mathcal{R}$.

Property LA.7.2 – Any unimodular matrix can be obtained from the identity matrix by a finite number of elementary row and column operations.

Property LA.7.3 – The product of two unimodular matrices is still a unimodular matrix.

Property LA.7.4 – Any elementary row (column) operation is equivalent to left (right) multiplication by a suitable unimodular matrix.

Property LA.7.5 – The inverses of unimodular matrices are still unimodular matrices.

Definition LA.7.4 – Two polynomial matrices $P(z)$ and $Q(z)$ are defined as row equivalent, column equivalent, or equivalent when one of them can be obtained from the other by a sequence of row, column or row and column operations respectively.

Definition LA.7.5 (Degree of a polynomial matrix) – The degree of a polynomial matrix $M(z)$ is defined as the degree of the polynomial of highest degree in $M(z)$. The degree of a row or of a column of $M(z)$ can be defined in the same way.

Definition LA.7.6 (Greatest common left (right) divisor) – If three polynomial matrices satisfy the relation $P(z) = H(z)G(z)$, then $G(z)$ $(H(z))$ is called a right (left) divisor of $P(z)$ and $P(z)$ is called a left (right) multiple of $G(z)$ $(H(z))$. A greatest common right divisor of (G.C.R.D.) of two polynomial matrices $P(z)$ and $R(z)$ is a common right divisor which is a left multiple of every common right divisor of $P(z)$ and $R(z)$. Similarly, a greatest common left divisor (G.C.L.D.) of two polynomial matrices $P(z)$ and $Q(z)$ is a common left divisor which is a right multiple of every common left divisor of $Q(z)$ and $P(z)$.

Property LA.7.6 – If a G.C.R.D. of two polynomial matrices is not a unimodular matrix, then a G.C.L.D. of these matrices is not necessarily a unimodular matrix and vice versa.

Definition LA.7.7 – Two polynomial matrices are defined as right prime (left prime) when their G.C.R.D. (G.C.L.D.) is a unimodular matrix.

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