ID6

ARMAX Identification

6.3 INSTRUMENTAL VARIABLE METHODS

Consider the application of the least squares estimate (3.3.12) to input–output sequences generated by an ARMAX process; denoting with e° the vector of equation errors (3.5.3) , the asymptotic behavior of the estimate is still described by relation (3.5.5) . Since the equation error of ARMAX models is not white, we have that

$$\text{plim}_{N\to\infty}\left[\,H^T e^\circ\,\right] \neq 0 \tag{6.3.1}$$

and, consequently, the LS estimate is biased. The bias can be approximated, for high values of N, by

$$b \simeq \left(H^T H\right)^{-1} H^T e^\circ. \tag{6.3.2}$$

Denote now with Z a matrix having the dimensions of H, i.e. $(N \times 2n)$, and with entries given by variables uncorrelated with equation errors $e(t)$; then

$$\mathrm{E}\left[\,Z^T e^\circ\,\right] = \mathrm{E}\left[\,Z^T\,\right]\mathrm{E}\left[\,e^\circ\,\right] = 0. \tag{6.3.3}$$

Substituting e° with the expression that can be deduced from (3.5.2) ,

$$e^\circ = y^\circ - H\theta^* \tag{6.3.4}$$

we obtain

$$\mathrm{E}\left[\,Z^T y^\circ - Z^T H\theta^*\,\right] = 0 \tag{6.3.5}$$

and, assuming $Z^T H$ $(2n \times 2n)$ as invertible, it is possible to deduce the unbiased estimate of θ^* given by

$$\theta^\circ = \left(Z^T H\right)^{-1} Z^T y^\circ. \tag{6.3.6}$$

Algorithm (6.3.6) can be considered as a generalization of least squares and is called Instrumental Variable method (IV); it was introduced by Reiersøl in 1941. It can be noted that expression (6.3.6) does not give any indication on the choice of the entries of

Z (instruments); the only conditions to be satisfied are (6.3.3) and the nonsingularity of $Z^T H$. A possible choice consists in taking a matrix of instruments given by

$$Z = \left[\, H_n(\eta)\ H_n(u) \,\right] \tag{6.3.7}$$

where $\eta(t)$ is generated by means of the difference equation

$$\eta(t) = a_{n_\eta}\eta(t-1) + \ldots + a_1\eta(t-n_\eta) + b_{n_\eta}u(t-1) + \ldots + b_1 u(t-n_\eta) \tag{6.3.8}$$

i.e. constitutes the output of a filter driven by the same input as the process to be identified. A very effective choice would be $n_\eta = n$, $a_i = \alpha_i$, $b_i = \beta_i$. Since true parameters α_i and β_i are not known, they can be initially substituted by a preliminary estimate (e.g. least squares) in order to get, using (6.3.6) a first IV estimate that can be used to obtain a second sequence of instruments leading to a second IV estimate and so on. The procedure can be repeated until convergence, usually reached in few iterations.

A second simple but less effective alternative consists in using as instruments past inputs, through relation

$$\eta(t) = u(t-n). \tag{6.3.9}$$

With this choice matrix Z is given by

$$Z = \begin{bmatrix} u(1-n) & \ldots & u(0) & u(1) & \ldots & u(n) \\ \vdots & & \vdots & \vdots & & \vdots \\ u(L-2n) & \ldots & u(L-n-1) & u(L-n) & \ldots & u(L-1) \end{bmatrix}. \tag{6.3.10}$$

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