ID3

ARX Identification

3.18 PARAMETRIC IDENTIFICATION OF MULTIVARIABLE ARX MODELS

The parametric identification of multivariable ARX models will rely on the minimality and identifiability of models (3.17.2) that, as already noted, define univocally ARX models (3.17.8) and (3.17.10). A specific model structure $\nu = (\nu_1, \ldots, \nu_m)$ will be considered; the structural identification is described in Module 3.20. With reference to model (3.17.2) consider the matrix

$$\begin{aligned} H_i &= H(\nu_{i1}, \ldots, \nu_i, \ldots, \nu_{im}) \\ &= \left[\, H_{\nu_{i1}}(y_1), \ldots, H_{\nu_i}(y_i), \ldots, H_{\nu_{im}}(y_m), H_{\nu_i}(u_1), \ldots, H_{\nu_i}(u_r) \,\right] \end{aligned} \tag{3.18.1}$$

and the vector

$$y_i^\circ = \left[\, y_i(\nu_i + 1) \ \ldots \ y_i(L) \,\right]^T. \tag{3.18.2}$$

Consider now the vector of parameters

$$\theta_i^\circ = \left[\alpha_{i11} \ldots \alpha_{i1\nu_{i1}} \mid \ldots \mid \alpha_{im1} \ldots \alpha_{im\nu_{im}} \mid \beta_{i11} \ldots \beta_{i1\nu_i} \mid \ldots \mid \beta_{ir1} \ldots \beta_{ir\nu_i}\,\right]^T \tag{3.18.3}$$

obtained by means of the least squares estimate

$$\theta_i^\circ = \left(H_i^T H_i\right)^{-1} H_i^T\, y_i^\circ \tag{3.18.4}$$

and the linear regression of input and output samples

$$y_i^p(\theta) = H_i\, \theta_i^\circ. \tag{3.18.5}$$

Because of the projection properties of least squares, θ_i° minimizes the Euclidean norm of

$$\varepsilon_i(\theta) = y_i^\circ - y_i^p(\theta). \tag{3.18.6}$$

The estimates θ_i° $(i = 1, \ldots, m)$ (3.18.4) define thus m linear regressions that minimize the cost function

$$J(\theta) = \sum_{i=1}^{m} \frac{\varepsilon_i^T(\theta)\,\varepsilon_i(\theta)}{N}. \tag{3.18.7}$$

Consider now the prediction error of predictor (3.17.11) when parameterized by the estimates θ_i

$$\varepsilon^*(t) = Q_{\nu_M+1}^{*-1}\varepsilon(t) \tag{3.18.8}$$

where

$$\varepsilon(t) = \left[\,\varepsilon_1(t)\;\ldots\;\varepsilon_m(t)\,\right]^T. \tag{3.18.9}$$

The lower left triangular structure of $Q_{\nu_M+1}^*$ with unitary elements on its main diagonal induces the same structure on its inverse; we can thus denote the entries of $Q_{\nu_M+1}^{*-1}$ as follows

$$Q_{\nu_M+1}^{*-1} = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ k_{21} & 1 & \ldots & 0 \\ \vdots & & & \vdots \\ k_{m1} & k_{m2} & \ldots & 1 \end{bmatrix}. \tag{3.18.10}$$

The components of $\varepsilon^*(t)$ are, consequently, given by

$$\begin{aligned} \varepsilon_1^*(t) &= \varepsilon_1(t) \\ \varepsilon_2^*(t) &= \varepsilon_2(t) + k_{21}\varepsilon_1(t) \\ &\ldots \\ \varepsilon_m^*(t) &= \varepsilon_m(t) + \ldots + k_{m1}\varepsilon_1(t)\,. \end{aligned} \tag{3.18.11}$$

As is shown in next section, the estimates θ_i° converge asymptotically to the true values of the model parameters so that also the vectors $\varepsilon_i(\theta)$ converge to the error vectors

$$e_i^\circ = \left[\,e_i(\nu_i + 1)\;\ldots\;e_i(L)\,\right]^T. \tag{3.18.12}$$

The asymptotic prevision error of predictor (3.17.11) is thus characterized by the errors

$$\varepsilon_i^*(t) = e_i(t) + k_{i(i-1)}e_{i-1}(t) + \ldots + k_{i1}e_1(t). \tag{3.18.13}$$

The cost function defined by the sum of the mean square errors of the single components of the prevision is

$$J^*(\theta) = \sum_{i=1}^{m} \frac{\varepsilon_i^{*T}(\theta)\,\varepsilon_i^*(\theta)}{N} \tag{3.18.14}$$

where $\varepsilon_i^*(\theta)$ is defined by (3.18.6) substituting $y_i^p(\theta)$ with the prevision $y_i^{*p}(\theta)$ given by predictor (3.17.11). Asymptotically we have, because of the independence of the sequences $e_i(t)$, that

$$\begin{aligned}\varepsilon_i^{*T}(\theta)\,\varepsilon_i^*(\theta) &= \big[\,e_i(\theta) + \ldots + k_{i1}e_1(\theta)\,\big]^T\big[\,e_i(\theta) + \ldots + k_{i1}e_1(\theta)\,\big] \\ &= e_i^T(\theta)\,e_i(\theta) + \ldots + k_{i1}^2 e_1^T(t)\,e_1(t)\end{aligned} \tag{3.18.15}$$

so that the LS estimates θ_i minimize the sum of the mean square prediction errors of the ARX predictor (3.17.11).