ID3

ARX Identification

3.4 IDENTIFIABILITY AND INPUT SELECTION

It has already been observed that the existence of a single parameterization θ°, compatible with the minimality of $J(\theta)$, is assured by the maximal rank of H or, equivalently, by the nonsingularity of $H^T H$. When this condition is not satisfied there exists an infinite number of parameterizations θ, corresponding to the same mean quadratic error $J(\theta)$; in such cases the solution given by (3.3.9) is *the* solution with minimal Euclidean norm because of the properties of pseudoinverses. It is thus possible to state that the maximal rank condition on H assures the identifiability of the parameters of ARX models. It is important to evaluate the conditions that assure in general, identifiability. A first condition concerns submatrix $H_n(u)$ which must have maximal rank otherwise also H would be rank–deficient. An input sequence, $u(\cdot)$, will be defined, to this respect, *persistently exciting* of order n in the interval $[1, L]$, when $H_n(u)$ has maximal rank, i.e. rank $H_n(u) = n$. More generally, it is possible to consider the following definition.

Definition 3.4.1 – Input sequence $u(\cdot)$ is defined as *persistently exciting* of order n when the matrix

$$\lim_{N\to\infty} \frac{H_n^T(u) H_n(u)}{N} \tag{3.4.1}$$

is nonsingular (positive definite).

It can be observed that $H_n^T(u)H_n(u)/N$ is under the assumption of zero mean value for $u(\cdot)$, the sample covariance matrix of this sequence and that expression (3.4.1) converges asymptotically, under previous assumption, to the covariance matrix of $u(\cdot)$. The suitability of a given sequence for identification purposes can be measured by the conditioning number of matrix (3.4.1) or of $H_n^T(u)H_n(u)/N$. In this context, an optimal sequence is given by white noise, that leads to orthogonal columns in $H_n(u)$; denoting with σ_u^2 its variance, the corresponding covariance matrix is given by $\sigma_u^2 I_n$. Other sequences of frequent use that approximate very well optimality conditions are Pseudo Random Binary Sequences (PRBS).

An input sequence persistently exciting of order n in the considered time interval constitutes a necessary but not sufficient condition for the independence of the columns of H. It can be easily verified that this condition is fulfilled if the number of columns of $H_n(y)$ is not greater than the order of the ARX process that has generated the sequences when the polynomials $q(z)$ and $p(z)$ are coprime. If, on the contrary, $q(z)$ and $p(z)$ have common factors, the same output sequence can be generated by a system with lower order. It is possible to conclude that the identifiability of an ARX model is assured when the input is persistently exciting of order n in the considered time interval and n is not greater than the minimal order of an ARX model compatible with the data.

Previous considerations can be easily extended to MISO models like (2.1.16) and (2.1.17) making reference, in defining persistently exciting inputs, to matrix

$$H_n(u) = \left[\, H_n(u_1) \ \dots \ H_n(u_r) \,\right] \tag{3.4.2}$$

and assuming the absence of common factors between $q(z)$ and the polynomials $p_i(z)$ $(i = 1, \dots, r)$. Also the extension to MIMO models (3.17.2) can be easily performed taking $n = \nu_M$ in (3.4.2).