ID6

ARMAX Identification

6.1 ARMAX MODELS

ARMAX models describe the equation error by means of a MA process of the type

$$e(t) = w(t) + \gamma_{n_\gamma} w(t-1) + \ldots + \gamma_1 w(t - n_\gamma) \tag{6.1.1}$$

where $w(\cdot)$ denotes a stochastic white process with null expected value, $\mathrm{E}\,[w(t)] = 0$. The whole ARMAX model has thus the following structure

$$\begin{aligned} y(t) = &\alpha_n y(t-1) + \ldots + \alpha_1 y(t-n) + \beta_n u(t-1) + \ldots + \beta_1 u(t-n) \\ &+ w(t) + \gamma_{n_\gamma} w(t-1) + \ldots + \gamma_1 w(t-n_\gamma)\,. \end{aligned} \tag{6.1.2}$$

Introducing the polynomial $r(z^{-1})$ and using the polynomials $q(z^{-1})$ (2.1.3) and $p(z^{-1})$ (2.1.4) ,

$$q(z^{-1}) = 1 - \alpha_n\, z^{-1} - \ldots - \alpha_1\, z^{-n} \tag{6.1.3}$$

$$p(z^{-1}) = \beta_n\, z^{-1} + \ldots + \beta_1\, z^{-n} \tag{6.1.4}$$

$$r(z^{-1}) = \gamma_{n_\gamma}\, z^{-1} + \ldots + \gamma_1\, z^{-n_\gamma}\,, \tag{6.1.5}$$

it is possible to rewrite model (6.1.2) in the compact form

$$q(z^{-1})\, y(t) = p(z^{-1})\, u(t) + r(z^{-1})\, w(t)\,. \tag{6.1.6}$$

The order, n_γ, of the MA part of the process is usually considered as potentially different from the order, n, of the autoregressive part; realization theory shows, however, that the order of every minimal state space realization of an ARMAX process is equal, when $q(z^{-1})$ and $r(z^{-1})$ are coprime, to max (n, n_γ), so that it is not restrictive to assume $n_\gamma = n$ as we will do in the following. Considering a forward notation, model (6.1.2) will be written as

$$\begin{aligned} y(t+n) = &\alpha_n y(t+n-1) + \ldots + \alpha_1 y(t) + \beta_n u(t+n-1) + \ldots + \beta_1 u(t) \\ &+ w(t+n) + \gamma_n w(t+n-1) + \ldots + \gamma_1 w(t). \end{aligned} \tag{6.1.7}$$

Introducing the polynomial $r(z)$ and using the polynomials $q(z)$ (2.1.14) and $p(z)$ (2.1.15)

$$q(z) = z^n - \alpha_n z^{n-1} - \ldots - \alpha_2 z - \alpha_1 \tag{6.1.8}$$

$$p(z) = \beta_n z^{n-1} + \ldots + \beta_2 z + \beta_1 \tag{6.1.9}$$

$$r(z) = z^n + \gamma_n z^{n-1} + \ldots + \gamma_2 z + \gamma_1 , \tag{6.1.10}$$

model (6.1.7) can be rewritten in the compact forms

$$q(z)\, y(t) = p(z)\, u(t) + r(z)\, w(t) \tag{6.1.11}$$

$$y(t) = \frac{p(z)}{q(z)} u(t) + \frac{r(z)}{q(z)} w(t) = G(z)\, u(t) + F(z)\, w(t). \tag{6.1.12}$$

Figure 6.1.1 shows the structure of ARMAX processes described by (6.1.12) .

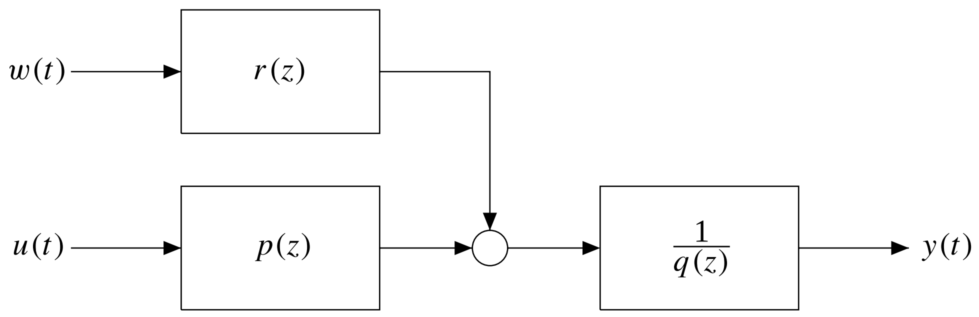

Figure 6.1.1 – Structure of an ARMAX process

Making reference to (6.1.12) it is also possible to partition an ARMAX process into deterministic and stochastic parts as shown in Figure 6.1.2 .

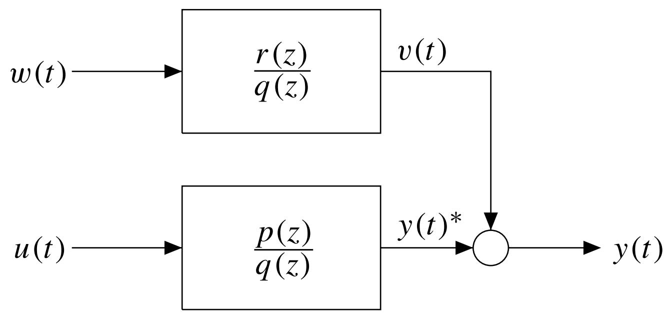

Figure 6.1.2 – Partition of an ARMAX process

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