ID7

ARMA Identification

7.6 EXAMPLE 7.2

We will deduce the optimal two–step–ahead predictor for ARMA process (7.4.1) whose polynomial form is

$$
\begin{aligned}
q(z^{-1}) &= 1 - 2\,z^{-1} + 1.68\,z^{-2} - 0.576\,z^{-3}\\
r(z^{-1}) &= 1 - 1.4\,z^{-1} + 0.59\,z^{-2} - 0.07\,z^{-3}.
\end{aligned}
$$

Algorithm (7.5.13) is initialized defining the coefficients of $a_1(z)$ and $b_1(z)$, given by

$$
\begin{aligned}
a_0^1 &= & 1\\
b_0^1 &= \gamma_3 + \alpha_3 = & 0.6\\
b_1^1 &= \gamma_2 + \alpha_2 = & -1.09\\
b_2^1 &= \gamma_1 + \alpha_1 = & 0.506;
\end{aligned}
$$

the optimal one–step–ahead predictor is thus

$$
\begin{aligned}
y(t+1|t) &= b_1(z^{-1})\,y(t) + \big(1 - r(z^{-1})\big)\,y(t+1|t)\\
&= 0.6\,y(t) - 1.09\,y(t-1) + 0.506\,y(t-2)\\
&\quad + 1.4\,y(t|t-1) - 0.59\,y(t-1|t-2) + 0.07\,y(t-2|t-3)
\end{aligned}
$$

and for $k = 2$ we obtain

$$
\begin{aligned}
a_0^2 &= a_0^1 = & 1\\
a_1^2 &= b_0^1 = & 0.6\\
b_0^2 &= b_1^1 + \alpha_3\,a_1^2 = & 0.11\\
b_1^2 &= b_2^1 + \alpha_2\,a_1^2 = & -0.502\\
b_2^2 &= b_3^1 + \alpha_1\,a_1^2 = & 0.3456.
\end{aligned}
$$

Note that in last relation it has been set $b_3^1 = 0$ since this parameter is not defined by previous relations. The optimal two–step–ahead predictor is thus given by

$$\begin{aligned} y(t+2|t) &= b_2(z^{-1})\, y(t) + \big(1 - r(z^{-1})\big)\, y(t+2|t) \\ &= 0.11\, y(t) - 0.502\, y(t-1) + 0.3456\, y(t-2) \\ &+ 1.4\, y(t+1|t-1) - 0.59\, y(t|t-2) + 0.07\, y(t-1|t-3). \end{aligned}$$

The two–step–ahead prediction given by this predictor is reported in Figure 7.6.1 (black line) where it is compared with the observed output. The residuals are reported in Figure 7.6.2.

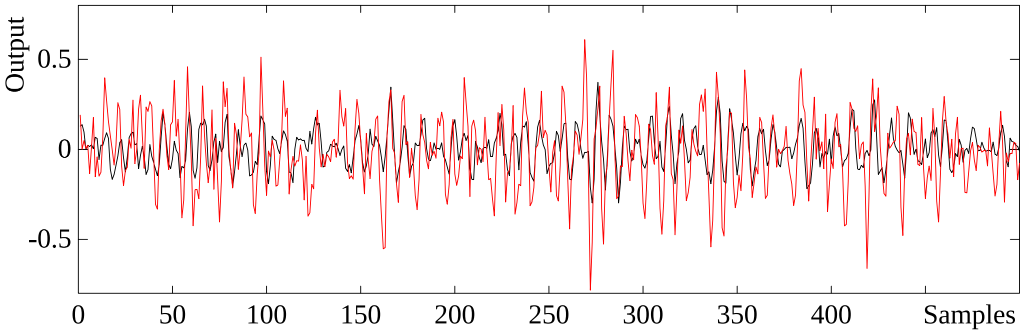

Figure 7.6.1 – Optimal two–step–ahead prevision (black line) and observed output

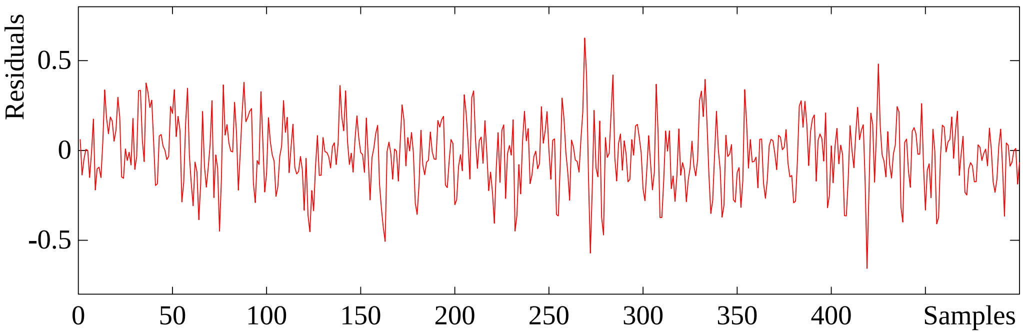

Figure 7.6.2 – Residuals of the two–step–ahead prevision

The variance of the prediction error is

$$\sigma_\varepsilon^2 = 0.0341\,,$$

not far from the value given by relation (7.5.11), equal to 0.0361.