ID10 ARARMA(X) Identification

10.4 PEM ESTIMATION OF ARARMAX MODELS

Considering parameterizations not far from the true one and approximating the residuals of the ARARMAX predictor with $w(t)$, we obtain, from (10.1.3), the expression

$$r(z^{-1})\,\varepsilon(t) = s(z^{-1})q(z^{-1})\,y(t) - s(z^{-1})\,p(z^{-1})\,u(t); \tag{10.4.1}$$

assuming $n_\delta = n$, the derivatives of $\varepsilon(t,\theta)$ are

$$\frac{\partial \varepsilon(t,\theta)}{\partial \alpha_i} = -\frac{s(z^{-1})}{r(z^{-1})}\,y(t+i-n-1) \tag{10.4.2}$$

$$\frac{\partial \varepsilon(t,\theta)}{\partial \beta_i} = -\frac{s(z^{-1})}{r(z^{-1})}\,u(t+i-n-1) \tag{10.4.3}$$

$$\frac{\partial \varepsilon(t,\theta)}{\partial \gamma_i} = -\frac{1}{r(z^{-1})}\,\varepsilon(t+i-n-1,\theta) \tag{10.4.4}$$

$$\begin{aligned}\frac{\partial \varepsilon(t,\theta)}{\partial \delta_i} &= -\frac{q(z^{-1})}{r(z^{-1})}\,y(t+i-n-1) + \frac{p(z^{-1})}{r(z^{-1})}\,u(t+i-n-1)\\ &= -\frac{1}{r(z^{-1})}\,e(t+i-n-1).\end{aligned} \tag{10.4.5}$$

By introducing the notations

$$y^F(t) = \frac{s(z^{-1})}{r(z^{-1})}\,y(t) \tag{10.4.6}$$

$$u^F(t) = \frac{s(z^{-1})}{r(z^{-1})}\,u(t) \tag{10.4.7}$$

$$\varepsilon^F(t) = \frac{1}{r(z^{-1})}\,\varepsilon(t) \tag{10.4.8}$$

$$e^F(t) = \frac{1}{r(z^{-1})}\, e(t), \tag{10.4.9}$$

we obtain for $\psi(t,\theta)$ (6.13.3) the following expression

$$\begin{aligned}\psi(t,\theta) = \big[\, & y^F(t-n)\ \ldots\ y^F(t-1)\ u^F(t-n)\ \ldots\ u^F(t-1) \\ & \varepsilon^F(t-n)\ \ldots\ \varepsilon^F(t-1)\ e^F(t-n)\ \ldots\ e^F(t-1)\,\big]^T.\end{aligned} \tag{10.4.10}$$

The Gauss–Newton algorithm

$$\theta^{k+1} = \theta^k + \left(H_\psi^T H_\psi\right)^{-1} H_\psi^T\, \varepsilon^\circ, \tag{10.4.11}$$

where

$$\theta^k = \left[\, \alpha_1^k\ \ldots\ \alpha_n^k\ \beta_1^k\ \ldots\ \beta_n^k\ \gamma_1^k\ \ldots\ \gamma_n^k\ \delta_1^k\ \ldots\ \delta_n^k \,\right]^T, \tag{10.4.12}$$

can be implemented by computing at every step, the matrix

$$H_\psi = \left[\begin{array}{cccccccccccc} y^F(1) & \ldots & y^F(n) & u^F(1) & \ldots & u^F(n) & \varepsilon^F(1) & \ldots & \varepsilon^F(n) & e^F(1) & \ldots & e^F(n) \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ y^F(L-n) & \ldots & y^F(L-1) & u^F(L-n) & \ldots & u^F(L-1) & \varepsilon^F(L-n) & \ldots & \varepsilon^F(L-1) & e^F(L-n) & \ldots & e^F(L-1) \end{array}\right] \tag{10.4.13}$$

and the vector

$$\varepsilon^\circ = \left[\, \varepsilon(n+1)\ \ldots\ \varepsilon(L) \,\right]^T. \tag{10.4.14}$$

Also in this case it can be useful to use, instead of (10.4.11), expressions of the type (6.13.19). The general case $n_\delta \neq n$ can be easily treated modifying (10.4.13) according to (9.5.11).