ID3

ARX Identification

3.19 BIAS AND CONSISTENCY OF MULTIVARIABLE LS ESTIMATES

To evaluate the properties of least squares estimates (3.18.4) we will start from the expression

$$y_i^\circ = H_i \theta_i^* + e_i^\circ \tag{3.19.1}$$

where θ_i^* denotes the vector of true parameters of the i–th subsystem of model (3.17.2). Following the same procedure already considered in Module 3.5 we obtain, from (3.18.4) and (3.19.1)

$$\begin{aligned}
\text{plim}_{N\to\infty}\theta_i^\circ &= \text{plim}_{N\to\infty}\left[\,(H_i^T H_i)^{-1} H_i^T (H_i\theta_i^* + e_i^\circ)\,\right] \\
&= \theta_i^* + \text{plim}_{N\to\infty}\left[\,(H_i^T H_i)^{-1} H_i^T e_i^\circ\,\right] \\
&= \theta_i^* + \text{plim}_{N\to\infty}\left[\left(\frac{H_i^T H_i}{N}\right)^{-1} \frac{H_i^T e_i^\circ}{N}\right] \\
&= \theta_i^* + \text{plim}_{N\to\infty}\left[\left(\frac{H_i^T H_i}{N}\right)^{-1}\right] \text{plim}_{N\to\infty}\left[\frac{H_i^T e_i^\circ}{N}\right].
\end{aligned} \tag{3.19.2}$$

Because of structure (3.18.1) of H_i, the product $H_i^T e_i^\circ$ has components given by the following terms

$$H_{\nu_{i1}}^T(y_1)\, e_i^\circ, \ldots, H_{\nu_{im}}^T(y_m)\, e_i^\circ, H_{\nu_i}^T(u_1)\, e_i^\circ, \ldots, H_{\nu_i}^T(u_r)\, e_i^\circ. \tag{3.19.3}$$

It is now easy to verify that the k–th element of $H_{\nu_{ij}}^T(y_j)\, e_i^\circ/N$ is given, for $i \neq j$, by

$$\left[\frac{H_{\nu_{ij}}^T(y_j)\, e_i^\circ}{N}\right]_k = \frac{1}{N}\sum_{s=0}^{N-1} y_j(k+s)\, e_i(\nu_i + s + 1) \qquad (k = 1, \ldots, \nu_{ij}) \tag{3.19.4}$$

and that

$$\text{plim}_{N\to\infty} \frac{1}{N} \sum_{s=0}^{N-1} y_j(k+s)\, e_i(\nu_i + s + 1) = 0 \qquad (k = 1, \dots, \nu_{ij}) \tag{3.19.5}$$

because of the independence of the errors $e_j(t)$ present on y_j from $e_i(t)$. For $i = j$ we obtain

$$\left[\frac{H_{\nu_i}^T(y_i)\, e_i^\circ}{N} \right]_k = \frac{1}{N} \sum_{s=0}^{N-1} y_i(k+s)\, e_i(\nu_i + s + 1) \qquad (k = 1, \dots, \nu_i) \tag{3.19.6}$$

and

$$\text{plim}_{N\to\infty} \frac{1}{N} \sum_{s=0}^{N-1} y_i(k+s)\, e_i(\nu_i + s + 1) = 0 \qquad (k = 1, \dots, \nu_i) \tag{3.19.7}$$

because of the whiteness of $e_i(t)$ and of the difference between the indexes of y_i and e_i in (3.19.7), always greater or equal to 1. Finally

$$\left[\frac{H_{\nu_i}^T(u_j)\, e_i^\circ}{N} \right]_k = \frac{1}{N} \sum_{s=0}^{N-1} u_j(k+s)\, e_i(\nu_i + s + 1) \qquad (k = 1, \dots, \nu_i) \tag{3.19.8}$$

and

$$\text{plim}_{N\to\infty} \frac{1}{N} \sum_{s=0}^{N-1} u_j(k+s)\, e_i(\nu_i + s + 1) = 0 \qquad (k = 1, \dots, \nu_i) \tag{3.19.9}$$

because of the independence between input sequences and equation errors. It follows that

$$\text{plim}_{N\to\infty} \theta_i^\circ = \theta_i^* \tag{3.19.10}$$

i.e. that the LS estimate of the multivariable ARX model (3.17.6) is unbiased.

Remark 3.19.1 – The whiteness of the components of $e(t)$ allows to establish, like in the scalar case, that

$$\lim_{N\to\infty} \left[\frac{H_i^T e_i^\circ}{N} \right] = 0 \tag{3.19.11}$$

with probability 1, i.e. that the obtained estimate is consistent.

Remark 3.19.2 – Note that the independence between the components of $e(t)$ plays an essential role in establishing the unbiasedness of multivariable LS estimates because in relation (3.19.4) ν_{ij} can be equal to $\nu_i + 1$. In this case (3.19.4) includes synchronous products $e_i(t)e_j(t)$ that allow to establish (3.19.6) only when $e_i(t)$ and $e_j(t)$ are independent.

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