ID3

ARX Identification

3.23 SENSITIVITY ANALYSIS OF IDENTIFIED MODELS

An important feature of identified models that is not detected by the validation procedures described in Module ID3.14 concerns the sensitivity of identified models to variations in their parameters. An high sensitivity to parameter variations can give rise to numerical problems in the use of the model for control and/or prediction. The sensitivity to variations of a single parameter, α_i, can be expressed by means of the function

$$S(\Delta\alpha_i) = \frac{\Delta J(\theta)\,\alpha_i^*}{J(\theta)\,\Delta\alpha_i} \tag{3.23.1}$$

where α_i^* is the estimated value of α_i, $J(\theta)$ the cost function corresponding to the identified parameter vector θ, $\Delta\alpha_i$ the considered perturbation of α_i^* and $\Delta J(\theta)$ the corresponding variation of the cost function. Function (3.23.1), which is nonlinear, can be computed over the desired range of parameter variations. It can also be of interest to consider the sensitivity to simultaneous variations in more than one parameter to evaluate possible synergies as shown in the following example.

Example – Reference will be made to the order 2 model identified in Example 3.15. The estimated values of its parameters are, for $N = 500$,

$$\begin{aligned}
\alpha_1 &= -0.3203\ (-0.34)\\
\alpha_2 &= 0.6016\ (0.6)\\
\beta_1 &= 0.2499\ (0.2738)\\
\beta_2 &= 0.4576\ (0.4564).
\end{aligned}$$

Introducing perturbations up to $\pm 20\%$ in the estimated values of α_1, α_2, β_1 and β_2 we obtain the values of $J(\theta)$ reported in Figure 3.23.1 where the red plot refers to α_1 and the blue one to α_2. The plots associated to variations of β_1 and β_2 are the magenta and green ones.

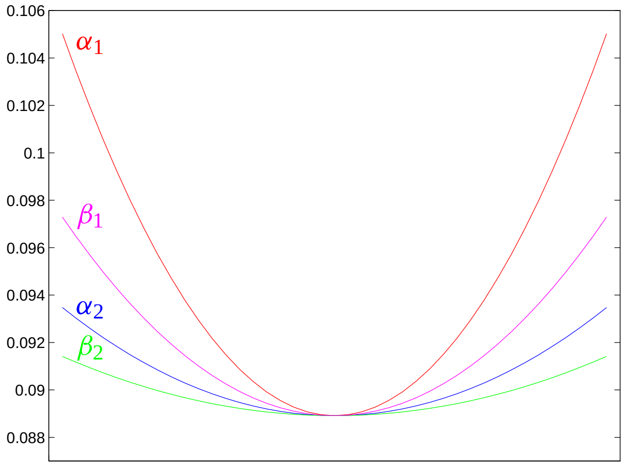


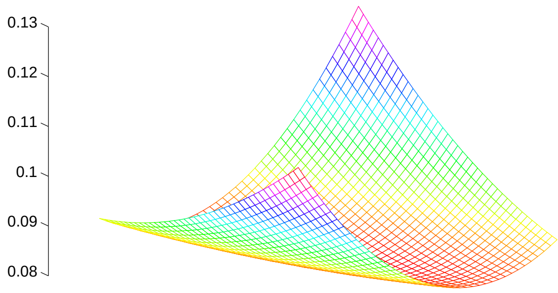
Figure 3.23.1 – Values of $J(\theta)$ for variations of $\pm 20\%$ in α_1, α_2, β_1 and β_2

These plots show that the maximal model sensitivity corresponds to variations of α_1, the minimal sensitivity to those of β_2. The standard deviations of the estimates of the model parameters are given by

$$\begin{aligned} \operatorname{std}\alpha_1 &= 0.024 & \qquad \operatorname{std}\beta_1 &= 0.034 \\ \operatorname{std}\alpha_2 &= 0.036 & \qquad \operatorname{std}\beta_2 &= 0.022; \end{aligned}$$

we can thus observe that the standard deviation of the estimate of α_1 is approximately equal to 7.5% of the value of this parameter so that the dispersion in the performance of the models that can be estimated in different experiments is modest. Similar considerations can be repeated for α_2, β_1 and β_2.

Considering simultaneous perturbations of α_1 and α_2 we obtain the surface reported in Figure 3.23.2 where, again, perturbations up to $\pm 20\%$ have been given to α_1 and α_2.

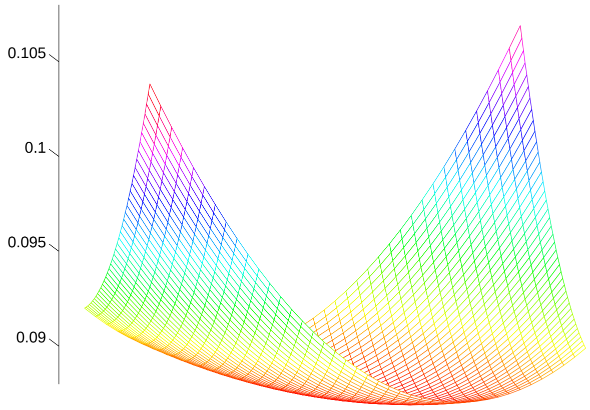


SURFACE ANALYSIS

Figure 3.23.2 – Values of $J(\theta)$ for variations of $\pm 20\%$ in α_1 and α_2

This surface shows the interaction between simultaneous variations in α_1 and α_2 and how these variations can lead to partial compensations or to cumulate negative influences on $J(\theta)$. A more detailed analysis can be performed using the button "Surface

Analysis" which loads a specific applet that allows to view the surface from any point of view.



SURFACE ANALYSIS

Figure 3.23.3 – Values of $J(\theta)$ for variations of $\pm 20\%$ in β_1 and β_2

Similar considerations can be repeated for the surface drawn in Figure 3.23.3 that refers to variations of β_1 and β_2. Also in this case a detailed analysis of the surface can be performed using the "Surface analysis" button.