ID5

MA Identification

5.2 IDENTIFICATION OF MA MODELS USING AUXILIARY AR MODELS

This identification procedure relies on the approximation of the MA process to be identified by means of an high–order AR model. High–order AR models can be used to approximate, with any desired degree of accuracy, MA models; in fact, substituting $w(t-1)$ with $y(t-1) - \gamma_n w(t-2) - \ldots - \gamma_1 w(t-n-1)$ in (5.1.1) , we obtain the equivalent model

$$y(t) = w(t) + \gamma_n y(t-1) - (\gamma_n^2 - \gamma_{n-1})\, w(t-2) - \ \ldots \ - \gamma_n \gamma_1 w(t-n-1). \quad (5.2.1)$$

Repeating this procedure it is possible to eliminate $w(t-2)$, $w(t-3)$, ...inserting autoregressive terms in $y(t-2)$, $y(t-3)$ and so on. Formally the procedure leads to insert an infinite number of autoregressive terms but with decreasing–value coefficients; it is thus possible to truncate this expansion leaving only the term $w(t)$ and a sufficient number of autoregressive terms $y(t-k)$ to achieve the desired approximation degree. Considering the available MA sequence as generated by an AR process, it is possible to use the identification techniques described for these processes to estimate a suitable order for the model and its parameters. The identified auxiliary AR model

$$y(t) = \alpha_{n_{AR}}\, y(t-1) + \ \ldots \ + \alpha_1\, y(t - n_{AR}) + e(t) \qquad (5.2.2)$$

allows to compute its residuals $e(t)$ which coincide with $w(t)$ (within the approximation introduced by representing the MA process with a finite–order AR one); these residuals can be used to estimate the parameters of the MA model

$$\theta = \left[\, \gamma_1 \ \ldots \ \gamma_n \,\right]^T \qquad (5.2.3)$$

using the least squares formula

$$\theta^\circ = \left(H^T H\right)^{-1} H^T y^\circ \qquad (5.2.4)$$

where

$$H = \begin{bmatrix} w(1) & w(2) & \dots & w(n) \\ w(2) & w(3) & \dots & w(n+1) \\ \vdots & \vdots & \dots & \vdots \\ w(N) & w(N+1) & \dots & w(N+n-1) \end{bmatrix} \tag{5.2.5}$$

and y° is given by (3.3.4) ; it is also possible, to take into account the finite length of the available sequences, to use the following expression for y°

$$y^\circ = \begin{bmatrix} y(n+1) - w(n+1) \\ y(n+2) - w(n+2) \\ \vdots \\ y(n+N) - w(n+N) \end{bmatrix} \tag{5.2.6}$$

which forces to 1 the coefficient of $w(t)$ in (5.1.1) ; the use of expressions (3.3.4) and (5.2.6) is equivalent when $w(\cdot)$ is white and N is large because the vector $[w(n+1), \dots, w(n+N)]^T$ is, under these assumptions, orthogonal to the columns of H.