ID3

ARX Identification

3.10 STATISTICAL PROPERTIES OF RESIDUALS

Consider prevision errors

$$\varepsilon(t) = y(t) - y(t|t-1) = h(t)\big(\theta^* - (H^T H)^{-1} H^T y^\circ\big) + e(t) \tag{3.10.1}$$

where $h(t)$ is defined by (3.6.5). It has already been shown that

$$\mathrm{plim}_{N\to\infty}\big[\theta^* - (H^T H)^{-1} H^T y^\circ\big] = 0; \tag{3.10.2}$$

it follows thus that prevision errors of ARX models estimated by least squares (or by means of other non biased algorithms) converge, for $N \to \infty$, to process $e(t)$. This property justifies whiteness tests on residuals to validate identified models. More useful information in validation and order selection is given by the variance of prediction errors, $\varepsilon(t)$, associated with the parameter estimate, θ° obtained from N samples, given by

$$\sigma_\varepsilon^2 = \mathrm{E}\left[\, \varepsilon^2(t, \theta_N^\circ)\,\right] \tag{3.10.3}$$

where notations θ_N° and $\varepsilon(t, \theta_N^\circ)$ underline the dependence of θ° on N and that of $\varepsilon(t)$ on θ_N°. It is possible to note that the expected value in (3.10.3) is conditioned by past values, i.e. by θ_N°. Considering large values of N and assuming as Gaussian the distribution of θ_N°, it is possible to prove that the expected value of σ_ε^2 is given by

$$\mathrm{E}\left[\,\sigma_\varepsilon^2\,\right] = \sigma_e^2\left(1 + \frac{d}{N}\right) \tag{3.10.4}$$

where $d = \dim\theta = 2n$. Relation (3.10.4) describes the dependence of the prediction error covariance on d/N and shows the disadvantages of overparameterized models; this property is usually referred to as the *parsimony principle*. It is also possible to show that an asymptotically unbiased estimate of σ_e^2 is given by

$$\hat{\sigma}_e^2 = \frac{1}{N-d}\sum_{t=n+1}^{L} \varepsilon(t)^2. \tag{3.10.5}$$

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