ID9

ARAR(X) Identification

9.6 EXAMPLE 9.1

The input–output sequences considered in this example are 520 samples generated by the the ARARX process

$$y(t) = 0.8\,y(t-1) - 0.32\,y(t-2) + u(t-1) + 0.8\,u(t-2) + e(t) \tag{9.6.1a}$$

$$e(t) = 0.2\,e(t-1) - 0.5\,e(t-2) + w(t). \tag{9.6.1b}$$

$w(t)$ is a white and Gaussian process with null expected value and sample variance (on 500 samples) $\sigma_w^2 = 1.0495$; the sample variance of the colored noise $v(t) = w(t)/(a(z)d(z))$ is $\sigma_v^2 = 2.4266$ while the sample variance of $y(t)^*$ is $\sigma_{y^*}^2 = 11.1341$. Previous figures can be interpreted as the presence of an amount of noise given by

$$100\,\frac{\sigma_v}{\sigma_{y^*}} = 46.7\%.$$

Estimating an ARX model with order 4 by means of least squares we obtain, for $N = 500$, the following parameters for $q(z^{-1})^*$ and $p(z^{-1})^*$

$$\begin{aligned}
\alpha_1^* &= -0.1617\ (-0.16) & \beta_1^* &= \ \ 0.4402\ (0.4)\\
\alpha_2^* &= \ \ 0.4583\ (0.464) & \beta_2^* &= \ \ 0.4109\ (0.34)\\
\alpha_3^* &= -0.9810\ (-0.98) & \beta_3^* &= \ \ 0.6519\ (0.6)\\
\alpha_4^* &= \ \ 0.9743\ (1) & \beta_4^* &= \ \ 0.9646\ (1).
\end{aligned}$$

The variance of the residuals of this model is $\sigma_\varepsilon^2 = 1.0433$, near to the true value; performing a whiteness test we obtain $\zeta_{500,8} = 4.3596$ and this value allows to consider the residuals as white. Performing an IV estimate using instruments generated with this model we obtain the following parameters α_i and β_i

$$\begin{aligned}
\alpha_1 &= -0.3274\ (-0.32) & \beta_1 &= \ \ 0.7465\ (0.8)\\
\alpha_2 &= \ \ 0.8222\ (0.8) & \beta_2 &= \ \ 1.0136\ (1).
\end{aligned}$$

It is now possible to compute the sequence of equation errors $e(t)$ (9.3.2) whose variance, given by $\sigma_{\hat{e}}^2 = 1.4681$, approximates well the true value $\sigma_e^2 = 1.4578$. Estimating now, with least squares, the AR model (9.3.3) we obtain the following parameters

$$\delta_1 = -0.5258\ (-0.5)$$
$$\delta_2 = \ \ 0.1662\ (0.2).$$

The variance of the residuals (9.3.4) of the whole ARARX model is $\sigma_\varepsilon^2 = 1.0475$, near to σ_w^2; the whiteness test gives $\zeta_{500,8} = 4.6977$ so that it is possible to conclude that the model interprets correctly the data.

This set of parameters has been used as initial estimate for the Gauss–Newton algorithm; (9.5.8) leads, after only three iterations, to the PEM estimate

$$\begin{aligned} \alpha_1 &= -0.3275\ (-0.32) & \beta_1 &= 0.8222\ (0.8) & \delta_1 &= -0.5196\ (-0.5) \\ \alpha_2 &= 0.8088\ (0.8) & \beta_2 &= 0.9801\ (1) & \delta_2 &= 0.1742\ (0.2). \end{aligned}$$

The mean square prevision error of this model is $J(\theta) = 1.0441$. The covariance matrix of the estimate (6.15.4) is

$$\operatorname{cov}\theta_{500}^{\circ} = 10^{-2}\begin{bmatrix} 0.161 & -0.223 & 0.202 & 0.118 & 0.010 & 0.182 \\ -0.223 & 0.360 & -0.444 & -0.154 & -0.040 & -0.283 \\ 0.202 & -0.444 & 1.193 & -0.216 & 0.092 & 0.330 \\ 0.118 & -0.154 & -0.216 & 0.499 & 0.008 & 0.125 \\ 0.010 & -0.040 & 0.092 & 0.008 & 0.161 & 0.011 \\ 0.182 & -0.283 & 0.330 & 0.125 & 0.011 & 0.370 \end{bmatrix};$$

the corresponding standard deviations of the parameter estimates are

$$\begin{aligned} \operatorname{std}\alpha_1 &= 0.040\ (0.0075) \\ \operatorname{std}\alpha_2 &= 0.060\ (0.0088) \\ \operatorname{std}\beta_1 &= 0.109\ (0.0222) \\ \operatorname{std}\beta_2 &= 0.071\ (0.0199) \\ \operatorname{std}\delta_1 &= 0.040\ (0.0196) \\ \operatorname{std}\delta_2 &= 0.061\ (0.0258) \end{aligned}$$

and it can be noted that actual deviations are lower than expected ones. The whiteness test on the residuals gives $\zeta_{500,8} = 4.8352$; the residuals must thus be considered as white. A further validation can be performed computing the correlation between the residuals and the input sequence $R_{\varepsilon u}^{500}(\tau)$ (3.14.25) reported in Figure 9.6.1 for $\tau = 0, \ldots, 8$. The horizontal lines denote confidence limits of 95% for a Gaussian distribution; the value of $\sigma_{\varepsilon u}^2$ has been computed using (3.14.26) with $k = -8, \ldots, 8$.

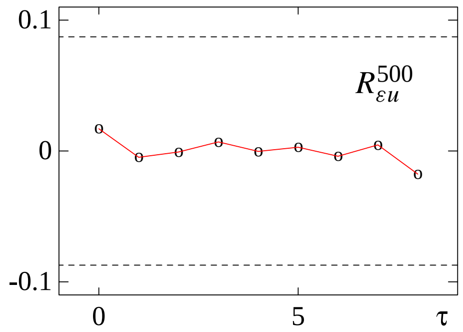

Figure 9.6.1 – Correlation between residuals and input

The one–step–ahead prevision obtained with the PEM model that has been estimated is reported in Figure 9.6.2 (black line) where it is compared with the observed output sequence. The corresponding residuals are reported in Figure 9.6.3.

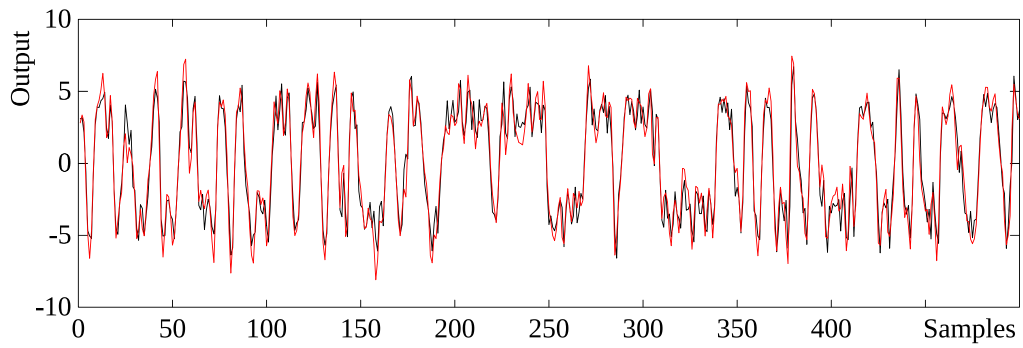

Figure 9.6.2 – Model prevision (black line) and observed output

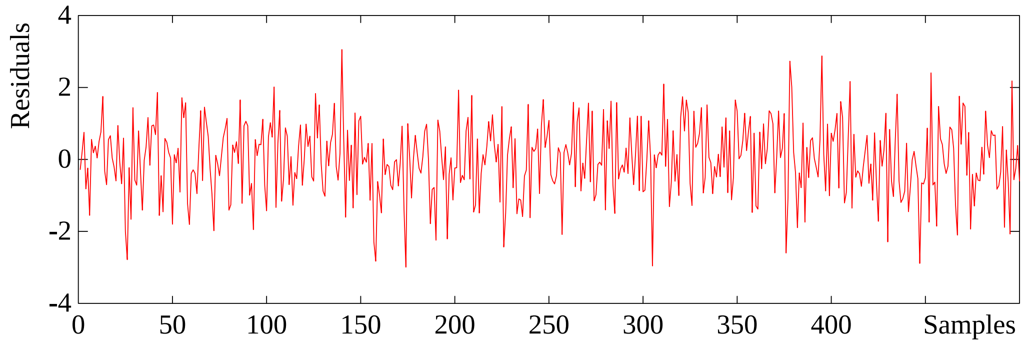

Figure 9.6.3 – Residuals of the ARARX PEM model