ID4

AR Identification

4.8 EXAMPLE 4.1

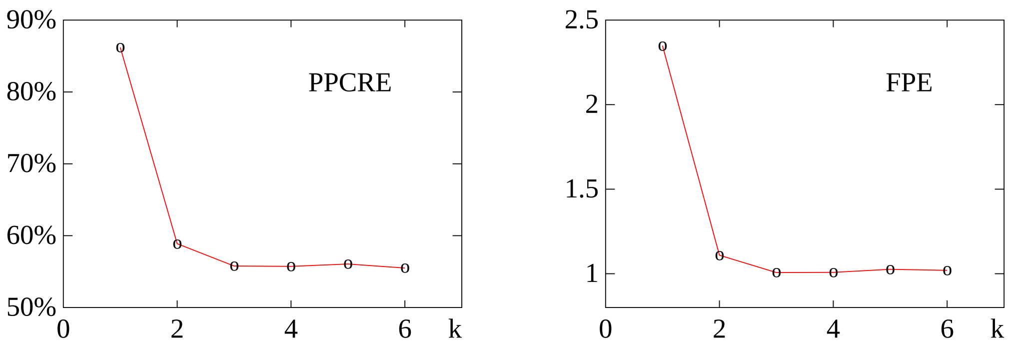
The AR process considered in this example is described by the model

$$y(t) = 1.2\,y(t-1) - 1.09\,y(t-2) + 0.438\,y(t-3) + e(t) \tag{4.8.1}$$

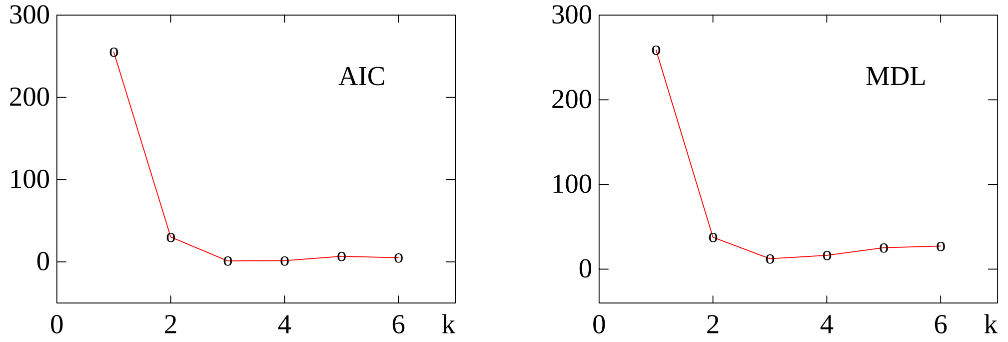
where $e(t)$ is a stationary and Gaussian process with null expected value and variance, computed on the first 300 samples, $\sigma_e^2 = 1$; it will be used to illustrate the application of the identification procedures described in previous sections.

4.8.1 Determination of the model order

The following criteria have been applied taking $N = 300$; the values of the PPCRE (3.14.14), computed for $k = 1, \ldots, 6$ are reported in Figure 4.8.1. A stabilization can be observed for $k > 3$ so that this criterion indicates 3 as most suitable order for the model. The results given by the FPE criterion are reported in Figure 4.8.2; also this criterion indicates 3 as correct order for the model.



Figures 4.8.1 and 4.8.2 – PPCRE and FPE criteria for $N = 300$



Figures 4.8.3 and 4.8.4 – AIC and MDL criteria for $N = 300$

The same indication can be deduced from AIC (Figure 4.8.3) and MDL (Figure 4.8.4) criteria. All criteria allow thus to deduce correctly the order of the model; using expression (3.14.16) it is possible to compute the corresponding variance of $e(\cdot)$ given, for $N = 300$, by

$$\hat{\sigma}_e^2 = 0.9970$$

which approximates very well the true value.

4.8.2 Parameter estimate

A first estimate has been obtained using the least squares algorithm (3.3.12) for $N = 300$. The obtained values are

$$\begin{aligned} \alpha_1 &= 0.3214 \ (0.438) \\ \alpha_2 &= -1.0122 \ (-1.09) \\ \alpha_3 &= 1.1209 \ (1.2) \end{aligned}$$

and the associated mean square prevision error is

$$J(\theta^\circ_{300}) = 0.9871\,.$$

The one–step–ahead output prevision (black line) is compared in Figure 4.8.5 with observed values; the residuals are plotted in Figure 4.8.6.

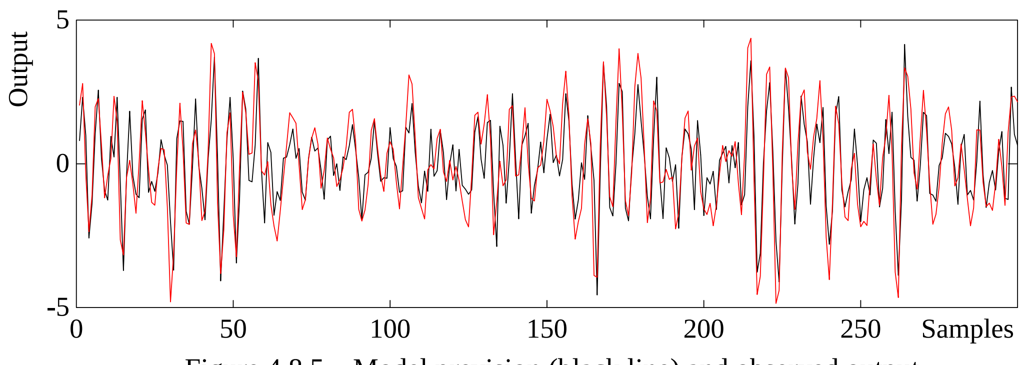


Figure 4.8.5 – Model prevision (black line) and observed output

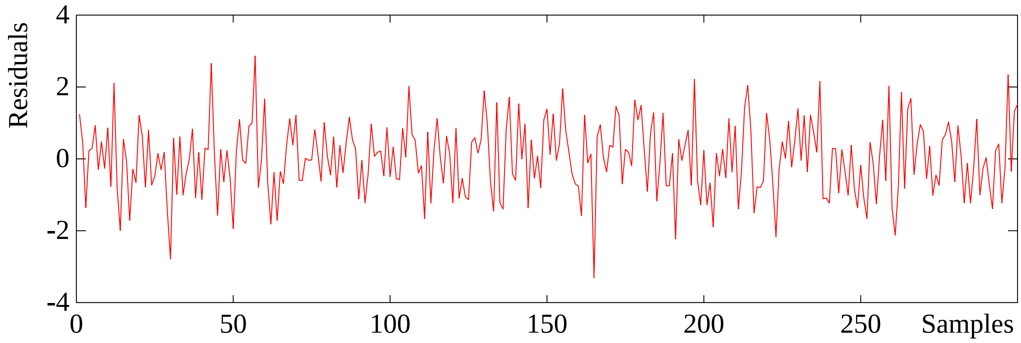


Figure 4.8.6 – Residuals of the model identified from 300 samples

The process has then been identified using Levinson algorithm to obtain a family of increasing–order models; this required computing, as first step, the following sample quantities

$$\hat{r}_i = \frac{1}{N} \sum_{t=1}^{N} y(t)\, y(t+i).$$

Taking $N = 300$ we obtain the following values

$$\begin{aligned}
\hat{r}_0 &= 3.1280 \\
\hat{r}_1 &= 1.5891 \\
\hat{r}_2 &= -0.8808 \\
\hat{r}_3 &= -1.5904 \\
\hat{r}_4 &= -0.4104 \\
\hat{r}_5 &= 0.8676 \\
\hat{r}_6 &= 0.8509 \\
\hat{r}_7 &= -0.2253 \\
\hat{r}_8 &= -1.0257 \\
\hat{r}_9 &= -0.7268 \\
\hat{r}_{10} &= 0.2826
\end{aligned}$$

The parameters obtained with Levinson algorithm (that will be denoted with double indexes to avoid any confusion between parameters belonging to different models) are:

$$\alpha_1^1 = \quad 0.5080$$

$$\alpha_1^2 = -0.7275$$
$$\alpha_2^2 = \quad 0.8776$$

$$\alpha_1^3 = \quad 0.3099 \quad (0.438)$$
$$\alpha_2^3 = -0.9995 \quad (-1.09)$$
$$\alpha_3^3 = \quad 1.1031 \quad (1.2)$$

$$\alpha_1^4 = -0.0293$$
$$\alpha_2^4 = \quad 0.3422$$
$$\alpha_3^4 = -1.0287$$
$$\alpha_4^4 = \quad 1.1121$$

$$\alpha_1^5 = \quad 0.0363$$
$$\alpha_2^5 = -0.0697$$
$$\alpha_3^5 = \quad 0.3796$$
$$\alpha_4^5 = -1.0411$$
$$\alpha_5^5 = \quad 1.1132$$

$$\alpha_1^6 = -0.0584$$
$$\alpha_2^6 = \quad 0.1013$$
$$\alpha_3^6 = -0.1305$$
$$\alpha_4^6 = \quad 0.4018$$
$$\alpha_5^6 = -1.0452$$
$$\alpha_6^6 = \quad 1.1153$$

It can be noted that Levinson algorithm leads, for $n = 3$, to parameters different from least squares ones. The reason is due to the necessity of assuming a Toeplitz structure in the moments matrix while this is not exactly true, for finite data lenghts, for $\hat{R}_n^s$ (4.5.13). This algorithm allows also computing at every step, the estimates of the variances of the equation errors corresponding to the orders considered; in this case we obtain

$$\sigma_{e1}^2 = 2.3206 \qquad \sigma_{e4}^2 = 0.9868$$
$$\sigma_{e2}^2 = 1.0926 \qquad \sigma_{e5}^2 = 0.9855$$
$$\sigma_{e3}^2 = 0.9876 \qquad \sigma_{e6}^2 = 0.9821$$

and it is possible to observe a stabilization, for $n > 3$, that gives the same information as PPCRE and allows to evaluate the correct model order. The same information can be deduced from partial correlation coefficients α_i^k.

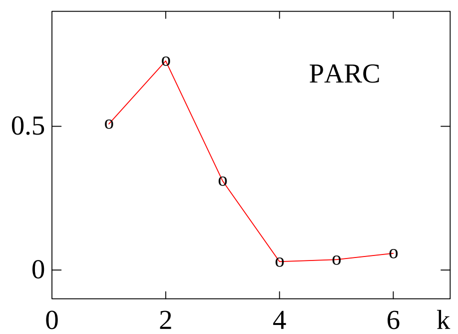


Figure 4.8.7 – Partial correlation coefficients (absolute values)

The plot of the absolute values of these quantities, reported in Figure 4.8.7, shows that they reach neglectable values for $k > 3$ and this confirms the choice $n = 3$ for the model order.

4.8.3 Model validation

A validation of the models that have been identified has been performed testing the whiteness of their residuals; computing $\zeta_{300,8}$ for the model estimated with least squares we obtain

$$\zeta_{300,8}^{LS} = 5.27$$

while the order 3 model obtained with Levinson gives

$$\zeta_{300,8}^{LV} = 5.39.$$

Since, for a confidence level of 99%, the value of χ^2_α, for $M = 8$, is $\chi^2_\alpha(8) = 20.1$, it is possible to conclude that the residuals of both models can be considered white. The comparison of the corresponding mean square prevision errors (0.9871 for least squares, 0.9874 for Levinson) confirms the essential equivalence of these models.

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