ID2

Equation error Identification

2.3 MULTIVARIABLE EQUATION ERROR MODELS

Equation error models for multi–input, multi–output (MIMO or multivariable) systems can be defined, similarly to the SISO and MISO cases, by means of relations of the type

$$y(t) = A_\mu y(t-1) + \ldots + A_1 y(t-\mu) + B_\mu u(t-1) + \ldots + B_1 u(t-\mu) + e(t) \quad (2.3.1)$$

where $u(t) \in \mathcal{R}^r$ and $y(t) \in \mathcal{R}^m$ are the input and output vectors, A_i and B_i are real $(m \times m)$ and $(m \times r)$ matrices and $e(t) \in \mathcal{R}^m$ is the equation error. Denoting with α_{ijk} and β_{ijk} the entries of A_i and B_i,

$$A_i = [\, \alpha_{ijk} \,] \qquad (j, k = 1, \ldots, m) \quad (2.3.2)$$

$$B_i = [\, \beta_{ijk} \,], \qquad (j = 1, \ldots, m), (k = 1, \ldots, r) \quad (2.3.3)$$

model (2.3.1) can also be written in the forms

$$y_i(t) = \sum_{j=1}^{m} \sum_{k=1}^{\mu} \alpha_{ijk}\, y_j(t + k - \mu - 1) + \sum_{j=1}^{r} \sum_{k=1}^{\mu} \beta_{ijk}\, u_j(t + k - \mu - 1) + e_i(t) \quad (i = 1, \ldots, m) \quad (2.3.4)$$

$$A(z^{-1})\, y(t) = B(z^{-1})\, u(t) + e(t) \quad (2.3.5)$$

where

$$A(z^{-1}) = [\, q_{ij}(z^{-1}) \,] = I - A_\mu z^{-1} - \ldots - A_1 z^{-\mu} \quad (2.3.6)$$

$$B(z^{-1}) = [\, p_{ij}(z^{-1}) \,] = B_\mu z^{-1} + \ldots + B_1 z^{-\mu}. \quad (2.3.7)$$

Considering a forward notation, models (2.3.1), (2.3.4) and (2.3.5) would be written as

$$y(t+\mu) = A_\mu y(t+\mu-1) + \ldots + A_1\, y(t) + B_\mu u(t+\mu-1) + \ldots + B_1 u(t) + e(t+\mu) \tag{2.3.8}$$

$$\begin{aligned} y_i(t+\mu) = &\sum_{j=1}^{m}\sum_{k=1}^{\mu} \alpha_{ijk}\, y_j(t+k-1) \\ &+ \sum_{j=1}^{r}\sum_{k=1}^{\mu} \beta_{ijk}\, u_j(t+k-1) + e_i(t+\mu) \quad (i=1,\ldots,m) \end{aligned} \tag{2.3.9}$$

$$A(z)\, y(t) = B(z)\, u(t) + z^\mu\, e(t) \tag{2.3.10}$$

where

$$A(z) = [\ q_{ij}(z)\ ] = I z^\mu - A_\mu z^{\mu-1} - \ \ldots\ - A_2 z - A_1 \tag{2.3.11}$$

$$B(z) = [\ p_{ij}(z)\ ] = B_\mu z^{\mu-1} + \ \ldots\ + B_2 z + B_1. \tag{2.3.12}$$

Previous models, that are commonly considered as the multivariable counterpart of SISO and MISO ones, could be used to define the multivariable equation error identification problem, in a way similar to the single–output case; the only difference would concern the meaning of the integer μ which defines the memory of the system and not its order. Their use in identification would not lead however, to efficient procedures because the memory and/or the order of a multivariable system do not give enough information on its internal structure to describe a minimally parameterized model. The use of models with non minimal parameterizations can in turn, lead to non identifiability conditions and, in any case, to increased uncertainties in the parameter estimates. We will thus make reference to models described by minimal parameterizations that can be written in the form

$$\begin{aligned} y_i(t+\nu_i) = &\sum_{j=1}^{m}\sum_{k=1}^{\nu_{ij}} \alpha_{ijk}\, y_j(t+k-1) \\ &+ \sum_{j=1}^{r}\sum_{k=1}^{\nu_i} \beta_{ijk}\, u_j(t+k-1) + e_i(t+\nu_i) \quad (i=1,\ldots,m) \end{aligned} \tag{2.3.13}$$

where the integers ν_i $(i = 1,\ldots,m)$ constitute the structure of the model and define the order of the system and its memory as

$$n = \sum_{i=1}^{m} \nu_i \tag{2.3.14}$$

$$\mu = \nu_M = \max_i \nu_i \,. \tag{2.3.15}$$

The integers ν_{ij} appearing in (2.3.13) are univocally defined by the structure of the model through the relations which follow

$$\nu_{ij} = \nu_i \qquad \text{for } i = j \tag{2.3.16a}$$

$$\nu_{ij} = \min\,(\nu_i + 1, \nu_j) \qquad \text{for } i > j \tag{2.3.16b}$$

$$\nu_{ij} = \min\,(\nu_i, \nu_j) \qquad \text{for } i < j \,. \tag{2.3.16c}$$

Model (2.3.13) can also be written in the equivalent polynomial form

$$Q(z)\, y(t) = P(z)\, u(t) + D(z)\, e(t) \tag{2.3.17}$$

where the entries of $Q(z)$ and $P(z)$ are given by

$$q_{ii}(z) = z^{\nu_i} - \alpha_{ii\nu_i}\, z^{(\nu_i - 1)} - \ldots - \alpha_{ii2} z - \alpha_{ii1} \tag{2.3.18a}$$

$$q_{ij}(z) = -\alpha_{ij\nu_{ij}}\, z^{(\nu_{ij} - 1)} - \ldots - \alpha_{ij2} z - \alpha_{ij1} \tag{2.3.18b}$$

$$p_{ij}(z) = \beta_{ij\nu_i}\, z^{(\nu_i - 1)} + \ldots + \beta_{ij2} z + \beta_{ij1} \tag{2.3.18c}$$

and $D(z)$ is the diagonal matrix

$$D(z) = \mathrm{diag}\left[\, z^{\nu_1}, \; \ldots, \; z^{\nu_m} \right]. \tag{2.3.19}$$

Remark 2.3.1 – Despite their apparent complexity, models (2.3.13) and (2.3.17) describe the input–output links established by multivariable systems in a very natural way and allow easy state space realizations. To realize the inadequacy of unstructured models in the identification of multivariable systems, consider for instance, a model with $m = 2$ and $\mu = 3$; generic values of the parameters α_{ijk} and β_{ijk} in models (2.3.10) would lead to an order $n = \deg \det\, Q(z) = \sum_{i=1}^{m} \mu = 6$. A system with two outputs and memory equal to 3 can however, also have order 4 and 5. These situations are described by model (2.3.10) only when suitable subsets of its parameters are null; this would happen with probability zero in identification so that the use of unstructured models with $m = 2$ and $\mu = 3$ would lead to identify order 6 models with probability one.