ID2

Equation error Identification

2.1 EQUATION ERROR MODELS

The most simple input–output relation that can be considered for linear, time–invariant, discrete–time and single–input single–output (SISO) systems, is the linear difference equation

$$y(t) = \alpha_n y(t-1) + \ldots + \alpha_1 y(t-n) + \beta_n u(t-1) + \ldots + \beta_1 u(t-n) \quad (2.1.1)$$

where $u(t)$ and $y(t)$ are the system input and output and n is the order of the model. Denoting with z the unitary advance operator (e.g., $z\,y(t) = y(t+1)$) and with z^{-1} the unitary delay operator, model (2.1.1) can be written in the more compact form

$$q(z^{-1})\,y(t) = p(z^{-1})\,u(t) \quad (2.1.2)$$

where

$$q(z^{-1}) = 1 - \alpha_n z^{-1} - \ldots - \alpha_1 z^{-n} \quad (2.1.3)$$

$$p(z^{-1}) = \beta_n z^{-1} + \ldots + \beta_1 z^{-n}. \quad (2.1.4)$$

Models of this type can be used in the realization of input–output sequences but not in identification applications where, even assuming a linear, finite–dimensional and time–invariant process behind the data, the presence of non measurable input(s) (disturbances) and of measurement errors, is not compatible with exact links like (2.1.1). The most simple way to take into account these deviations consists in describing their resulting effect by means of an error term, $e(t)$; this leads to *equation error models* of the type

$$y(t) = \alpha_n y(t-1) + \ldots + \alpha_1 y(t-n) + \beta_n u(t-1) + \ldots + \beta_1 u(t-n) + e(t) \quad (2.1.5)$$

or, equivalently,

$$q(z^{-1})\,y(t) = p(z^{-1})\,u(t) + e(t). \quad (2.1.6)$$

In equation error identification, equation errors are described by means of stochastic processes constituted by white noises (ARX and AR models), moving averages of white noises (ARMAX, ARMA, ARIMAX and ARIMA models), autoregressions of white noises (ARARX and ARAR models), ARMA processes (ARARMAX and ARARMA models) or by the output of filters driven by white noises (Box–Jenkins models).

Remark 2.1.1 – The notation used in (2.1.1) ad in all equation error models that follow can look unnatural and does not coincide with usual notations that are of the type

$$y(t) = \alpha_1 y(t-1) + \ldots + \alpha_n y(t-n) + \beta_1 u(t-1) + \ldots + \beta_n u(t-n). \quad (2.1.7)$$

Notation (2.1.7) is absolutely natural as long as "backward" single–output or multi–output non canonical models are considered, but its extensions to describe canonical input–output and state–space multivariable models are unnecessarily complicated. Since the treatment of multivariable identification is an essential part of this course, it has been considered as preferable to adopt a uniform notation oriented at the description of the most complex environments. The alternative of using different notations for different models has been considered as not advisable because it would prevent the direct deduction of SISO and MISO models as particular cases of MIMO ones. Moreover the notation that has been adopted is more natural than standard one in "forward" models (see Remark 2.1.4).

Remark 2.1.2 – Models (2.1.1), (2.1.2), (2.1.5) and (2.1.6) are *purely dynamic* since the output at time t is not affected by the value of the input at the same time. The extension of equation error identification procedures to non purely dynamic models is avoided because of the prevalent use of these models for prediction and control.

Remark 2.1.3 – Equation error models are usually written in the form

$$y(t)+\alpha_1 y(t-1)+\ldots+\alpha_{n_\alpha} y(t-n_\alpha) = \beta_1 u(t-1)+\ldots+\beta_{n_\beta} u(t-n_\beta)+e(t) \quad (2.1.8)$$

which, besides the differences described in Remark 2.1.1 and the irrelevant change of sign in the α_i parameters, differs from (2.1.5) because of different memory values, n_α and n_β, in the autoregressive and moving average parts of the model. Different values for n_α and n_β introduce, apparently, a further degree of freedom in the model. Making reference to the most general class of models for dynamic systems, i.e. state–space ones, we see, however, that assuming $q(z^{-1})$ and $p(z^{-1})$ as coprime, any minimal state–space realization of model (2.1.8) has order $n = \max(n_\alpha, n_\beta)$. Different values for n_α and n_β correspond thus to *a priori* assumptions on the values of the parameters. This practice derives from the introduction of equation error models *before* the introduction of state–space models and realization theory. In the following we will always assume $n_\alpha = n_\beta = n$.

Previous SISO models can be extended, in a straightforward way, to multi–input single–output (MISO) systems, defining input–output relations of the type

$$y(t) = \sum_{i=1}^{n} \alpha_i \, y(t+i-n-1) + \sum_{i=1}^{r} \sum_{j=1}^{n} \beta_{ij} \, u_i(t+j-n-1) + e(t) \tag{2.1.9}$$

or

$$q(z^{-1}) \, y(t) = \sum_{i=1}^{r} p_i(z^{-1}) \, u_i(t) + e(t) \tag{2.1.10}$$

where r denotes the number of inputs and

$$p_i(z^{-1}) = \beta_{in} z^{-1} + \ldots + \beta_{i1} \, z^{-n} \, . \tag{2.1.11}$$

Remark 2.1.4 – Alternative representations of models (2.1.5) and (2.1.9) rely on forward instead than backward time notations. Model (2.1.5) can thus be written in the form

$$y(t+n) = \alpha_n \, y(t+n-1) + \ldots + \alpha_1 \, y(t) + \beta_n u(t+n-1) + \ldots + \beta_1 u(t) + e(t+n) \tag{2.1.12}$$

or

$$q(z) \, y(t) = p(z) \, u(t) + z^n \, e(t) \tag{2.1.13}$$

where

$$q(z) = z^n - \alpha_n z^{n-1} - \ldots - \alpha_2 z - \alpha_1 \tag{2.1.14}$$

$$p(z) = \beta_n z^{n-1} + \ldots + \beta_2 z + \beta_1 \, . \tag{2.1.15}$$

Model (2.1.9) can be written as

$$y(t+n) = \sum_{i=1}^{n} \alpha_i \, y(t+i-1) + \sum_{i=1}^{r} \sum_{j=1}^{n} \beta_{ij} \, u_i(t+j-1) + e(t+n) \tag{2.1.16}$$

or

$$q(z) \, y(t) = \sum_{i=1}^{r} p_i(z) \, u_i(t) + z^n e(t) \tag{2.1.17}$$

with

$$p_i(z) = \beta_{in} \, z^{n-1} + \ldots + \beta_{i2} \, z + \beta_{i1} . \tag{2.1.18}$$

These notations are equivalent to previous ones; it must only be remembered that the asymptotic stability condition for models (2.1.13) and (2.1.17) requires all zeros of

$q(z)$ (system poles) inside the unit circle while the condition for models (2.1.6) and (2.1.10) requires all zeros of $q(z^{-1})$ outside this circle.

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