ID4

AR Identification

4.4 ESTIMATION OF INCREASING–ORDER MODELS

It is sometimes useful to dispose of the parameter estimates for a whole family of increasing–order AR models obtained from the same input–output sequence. In solving this problem it is possible to avoid recomputing for every new model, the inverse of matrix $S = H^T H$ i.e. the most demanding computation; consider, for this purpose, the parameter estimate for a model with order $k+1$, given by

$$\theta^\circ_{k+1} = \left(H^T_{k+1} H_{k+1}\right)^{-1} H^T_{k+1}\, y^\circ_{k+1} \tag{4.4.1}$$

where $H_{k+1} = H_{k+1}(y)$ and $y^\circ_{k+1} = [y(k+2) \ldots y(k+N+1)]^T$. Moreover

$$H_{k+1} = \begin{bmatrix} H_k & y^\circ_k \end{bmatrix} \tag{4.4.2}$$

$$S_{k+1} = H^T_{k+1} H_{k+1} = \begin{bmatrix} S_k & H^T_k\, y^\circ_k \\ y^{\circ T}_k H_k & y^{\circ T}_k y^\circ_k \end{bmatrix}. \tag{4.4.3}$$

S_{k+1} can thus be obtained bordering S_k with a row and a column. Using a well known lemma on the inversion of partitioned matrices we obtain

$$\begin{aligned} S^{-1}_{k+1} &= \begin{bmatrix} S^{-1}_k & 0 \\ 0 & 0 \end{bmatrix} \\ &\quad + \begin{bmatrix} -S^{-1}_k H^T_k\, y^\circ_k \\ 1 \end{bmatrix} \begin{bmatrix} y^{\circ T}_k y^\circ_k - y^{\circ T}_k H_k\, S^{-1}_k H^T_k\, y^\circ_k \end{bmatrix}^{-1} \begin{bmatrix} -y^{\circ T}_k H_k\, S^{-1}_k & 1 \end{bmatrix}; \end{aligned}$$

considering that $\theta^\circ_k = S^{-1}_k H^T_k\, y^\circ_k$, and denoting with σ^2 the expression

$$\sigma^2 = y^{\circ T}_k \left(y^\circ_k - H_k \theta^\circ_k\right), \tag{4.4.4}$$

the inverse of S_{k+1} can be rewritten in the form

$$S_{k+1}^{-1} = \frac{\begin{bmatrix} \sigma^2 S_k^{-1} + \theta_k^\circ \theta_k^{\circ T} & -\theta_k^\circ \\ -\theta_k^{\circ T} & 1 \end{bmatrix}}{\sigma^2}. \tag{4.4.5}$$

Relation (4.4.5) allows estimating the parameter vectors $\theta_k^\circ, \theta_{k+1}^\circ, \ldots$ i.e. the parameters of increasing–order models using, at every step, previous estimates.

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