ID3

ARX Identification

3.1 ARX MODELS

ARX models are the most simple member of the equation error family; they describe equation errors by means of white processes. Making reference to SISO model (2.1.5)

$$y(t) = \alpha_n\, y(t-1) + \ldots + \alpha_1\, y(t-n) + \beta_n u(t-1) + \ldots + \beta_1 u(t-n) + e(t), \tag{3.1.1}$$

$e(\cdot)$ will be a stochastic white process with null expected value, $\mathrm{E}\,[e(t)] = 0$, that is assumed as independent of the input sequence $u(\cdot)$. The integer n defines the order and the memory of the model. The letters AR in the acronym ARX denote the autoregressive part of the model, while X refers to the presence of an input i.e., derived from a term used in econometrics, eXogenous variable (other interpretations derive the X from eXternal).

Model (3.1.1) can be easily extended to the case of processes where the input acts with a delay, n_d, equal to a finite number of sampling intervals; the corresponding ARX model will be

$$\begin{aligned} y(t) &= \alpha_n\, y(t-1) + \ \ldots\ + \alpha_1\, y(t-n) \\ &+ \beta_n u(t - n_d - 1) + \ \ldots\ + \beta_1 u(t - n_d - n) + e(t)\,. \end{aligned} \tag{3.1.2}$$

It is common to consider ARX models of type (2.1.8), i.e. with different values of the memory associated with past inputs and outputs; since this does not introduce any generalization or advantage, we will deviate from tradition using only models of type (3.1.1). Considering the polynomial form (2.1.6)

$$q(z^{-1})\, y(t) = p(z^{-1})\, u(t) + e(t) \tag{3.1.3}$$

where

$$q(z^{-1}) = 1 - \alpha_n\, z^{-1} - \ldots - \alpha_1\, z^{-n} \tag{3.1.4}$$

$$p(z^{-1}) = \beta_n\, z^{-1} + \ldots + \beta_1\, z^{-n}\,, \tag{3.1.5}$$

it is possible to rewrite model (3.1.1) in the significant form

$$y(t) = \frac{p(z^{-1})}{q(z^{-1})}\, u(t) + \frac{1}{q(z^{-1})}\, e(t) = G(z^{-1})\, u(t) + F(z^{-1})\, e(t) \tag{3.1.6}$$

or, using forward notation (2.1.13), in the more common form

$$y(t) = \frac{p(z)}{q(z)}\, u(t) + \frac{z^n}{q(z)}\, e(t) = G(z)\, u(t) + F(z)\, e(t)\,. \tag{3.1.7}$$

Figure 3.1.1 shows the structure of ARX processes described by (3.1.7).

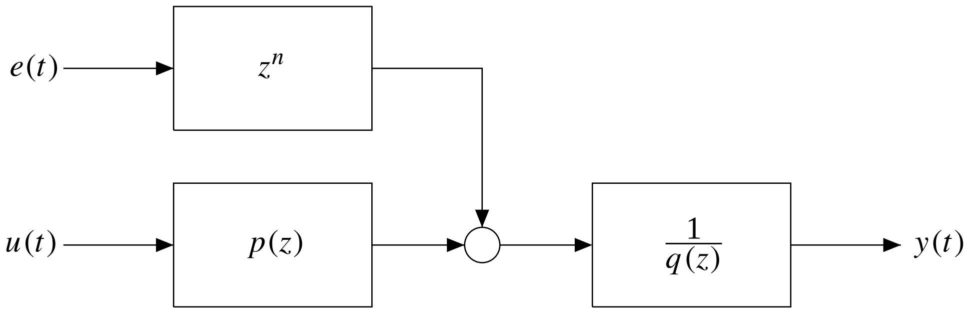

Figure 3.1.1 – Structure of an ARX process

Making reference to (3.1.7) it is also possible to partition an ARX process into deterministic and stochastic parts as shown in Figure 3.1.2.

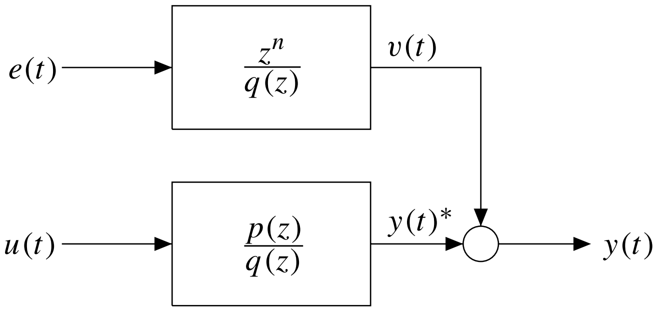

Figure 3.1.2 – Partition of an ARX process