ID6

ARMAX Identification

6.10 RECURSIVE IV ALGORITHMS

Consider expression (6.3.6) of the IV estimator; evidentiating the dependence from time $t = L$ of the estimate θ°, of the instrument matrix Z, of the samples matrix H and of vector y°, we can write expression

$$\theta^\circ(t) = \left(Z^T(t)\, H(t)\right)^{-1} Z^T(t)\, y^\circ(t), \tag{6.10.1}$$

similar to (3.6.1) . It is possible to derive a recursive version of the IV algorithm along the same line already followed for least squares, obtaining also in this case the update expression

$$\theta^\circ(t) = \theta^\circ(t-1) + K(t)\, e(t) \tag{6.10.2}$$

where the gain matrix $K(t)$ is given by

$$K(t) = \frac{R(t)^{-1} z^T(t)}{N} \tag{6.10.3}$$

and the equation error $e(t)$ by

$$e(t) = y(t) - h(t)\, \theta(t-1) \tag{6.10.4}$$

with

$$h(t) = \left[\, y(t-n)\ \ldots\ y(t-1)\, u(t-n)\ \ldots\ u(t-1)\,\right]. \tag{6.10.5}$$

Moreover

$$R(t) = \frac{Z^T(t)\, H(t)}{N} \tag{6.10.6}$$

while $z(t)$ denotes the last row of the instrument matrix $Z(t)$ i.e. the row that must be added to $Z(t-1)$ to obtain $Z(t)$. The update of $R(t)^{-1}$ is given by

$$R(t)^{-1} = \frac{N\, R(t-1)^{-1}}{N-1} \left[I - \frac{z^T(t)\, h(t)\, R(t-1)^{-1}}{N - 1 + h(t)\, R(t-1)^{-1} z^T(t)} \right]. \tag{6.10.7}$$

As with recursive least squares, the algorithm will start from an initial IV estimate $\theta^{\circ}(t_0)$ and from the computation of $R(t_0)^{-1}$; the update of these quantities will then rely, at every step, on expressions (6.10.2) and (6.10.7) .