ID9
ARAR(X) Identification

9.1 ARAR(X) MODELS

ARARX models, sometimes denoted also as ARXAR, describe the equation error by means of the AR process

$$e(t) = \delta_{n_\delta} e(t-1) + \ldots + \delta_1 e(t-n_\delta) + w(t) \tag{9.1.1}$$

where $w(t)$ is a remote white process with null expected value, $\mathrm{E}[w(t)] = 0$, independent from the input sequence, $u(\cdot)$. A single–input single–output ARARX model is given by relation (9.1.1) and by the usual equation error model

$$y(t) = \alpha_n y(t-1) + \ldots + \alpha_1 y(t-n) + \beta_n u(t-1) + \ldots + \beta_1 u(t-n) + e(t). \tag{9.1.2}$$

Introducing the polynomial

$$s(z^{-1}) = 1 - \delta_{n_\delta} z^{-1} - \ldots - \delta_1 z^{-n_\delta}, \tag{9.1.3}$$

an ARARX model can be written in the polynomial form

$$q(z^{-1})\, y(t) = p(z^{-1})\, u(t) + \frac{w(t)}{s(z^{-1})} \tag{9.1.4}$$

where $q(z^{-1})$ and $p(z^{-1})$ are defined by (6.1.3) and (6.1.4). Defining $s(z)$ as

$$s(z) = z^{n_\delta} - \delta_{n_\delta}\, z^{n_\delta - 1} - \ldots - \delta_1 \tag{9.1.5}$$

and multiplying both members of (9.1.4) for z^{n^*}, with $n^* = n + n_\delta$, we can rewrite the model in forward notation as

$$y(t) = \frac{p(z)}{q(z)}\, u(t) + \frac{z^{n^*}}{q(z)s(z)}\, w(t) = G(z)\, u(t) + F(z)\, w(t) \tag{9.1.6}$$

where $q(z)$ and $p(z)$ are defined by (6.1.8) and (6.1.9). Figure 9.1.1 shows the structure of ARARX processes described by (9.1.6).

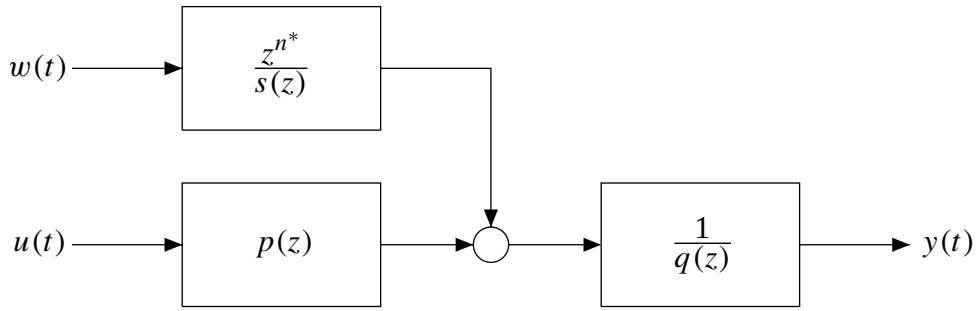

Figure 9.1.1 – Structure of an ARARX process

Structure (9.1.6) of ARARX models shows that also these models can be decomposed, according to the block diagram of Figure 9.1.2, into a deterministic part and a stochastic one given by a filter driven by remote white noise.

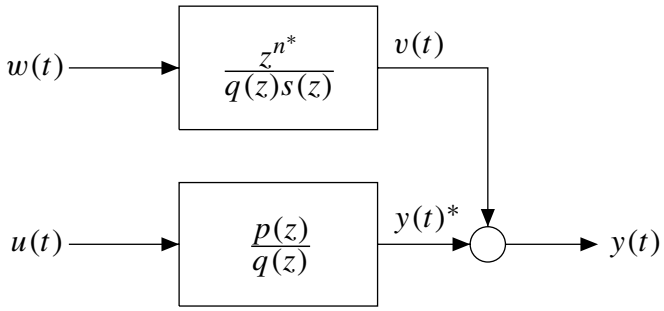

Figure 9.1.2 – Partition of an ARARX process

Remark 9.1.1 – In the case of ARARX models there are no obvious relations between n and n_δ and every minimal state space realization will have an order equal to n^*. The assumption $n_\delta = n$ is, however, not infrequent.