ID6

ARMAX Identification

6.7 ASYMPTOTIC PROPERTIES OF IV ESTIMATES

It has already been observed that the consistency of IV estimates derives from condition (6.3.3) which assures that

$$\operatorname{plim}_{N\to\infty} b = 0. \tag{6.7.1}$$

It is, however, also necessary that $Z^T H$ be a nonsingular matrix; it is thus necessary (but not sufficient) that

$$\operatorname{rank} Z = 2n. \tag{6.7.2}$$

Previous conditions require that the instruments be uncorrelated with equation errors but correlated with input sequence $u(t)$ and with the output, $y(t)^*$, of the deterministic part of the process; hence the choice of filter (6.3.8) to generate the instruments or to the choice of past inputs as instruments. In this last case the amount of delay must be sufficient to avoid any correlation with $e(t)$ but should not be excessive in order to guarantee an high correlation with $y(t)^*$.

All admissible choices of instruments lead to unbiased estimates but characterized by different covariance matrices; it is possible to prove that the distribution of the random variable

$$\sqrt{N}\left(\theta^\circ - \theta^*\right) \tag{6.7.3}$$

converges, for $N \to \infty$, to a zero–mean Gaussian distribution. The covariance matrix of the estimate is given by

$$\operatorname{cov}\theta^\circ = \sigma_w^2 \ \mathrm{E}\left[\left(Z^T H\right)^{-1} Z^{*T} Z^* \left(H^T Z\right)^{-1}\right] \tag{6.7.4}$$

where σ_w^2 denotes the variance of $w(t)$ and

$$Z^* = \left[\, z_{ij}^*(t) \,\right] \qquad (i = 1, \ldots, N)\ (j = 1, \ldots, 2n) \tag{6.7.5}$$

$$z_{ij}^*(t) = r(z^{-1})\, z_{ij}(t), \tag{6.7.6}$$

having denoted with $z_{ij}(t)$ the entries of Z. Relation (6.7.4) can, obviously, be approximated as

$$\operatorname{cov}\theta^\circ = \sigma_w^2\left(Z^T H\right)^{-1} Z^{*T} Z^*\left(H^T Z\right)^{-1}. \tag{6.7.7}$$