ID5

MA Identification

5.8 MULTIVARIABLE MA MODELS

Differently from AR models, it is not possible to define minimal parameterizations for single MA models. They can be defined, in general, by the relation

$$y(t) = w(t) + R_\mu \, w(t-1) + \ldots + R_1 \, w(t-\mu) \tag{5.8.1}$$

where $y(t) \in \mathcal{R}^m$, $w(t) \in \mathcal{R}^m$ and the components of $w(t)$ are white processes. The equivalent forward notation is

$$z^\mu \, y(t) = R(z) \, w(t) \tag{5.8.2}$$

where

$$R(z) = I \, z^\mu + R_\mu z^{\mu-1} + \ldots + R_2 \, z + R_1 = \left[\, r_{ij}(z) \, \right] \quad (i, j = 1, \ldots, m) \tag{5.8.3}$$

$$r_{ii}(z) = z^\mu + \gamma_{ii\mu} \, z^{\mu-1} + \ldots + \gamma_{ii2} \, z + \gamma_{ii1} \tag{5.8.4a}$$

$$r_{ij}(z) = \gamma_{ij\mu} \, z^{\mu-1} + \ldots + \gamma_{ij2} \, z + \gamma_{ij1}. \tag{5.8.4b}$$

The estimation of these models can be performed estimating $w(t)$ as residual of an auxiliary high order AR model and defining the matrix

$$H = \left[\, H_\mu(w_1) \, \ldots \, H_\mu(w_m) \, \right] \tag{5.8.5}$$

and the vector

$$y_i^\circ = \begin{bmatrix} y_i(\mu+1) - w_i(\mu+1) \\ y_i(\mu+2) - w_i(\mu+2) \\ \vdots \\ y_i(\mu+N) - w_i(\mu+N) \end{bmatrix}. \tag{5.8.6}$$

The LS estimate of the vector of parameters

$$\theta_i^\circ = \left[\, \gamma_{i11} \ \dots \ \gamma_{i1\mu} \ \dots \ \gamma_{ii1} \ \dots \ \gamma_{ii\mu} \ \dots \ \gamma_{im1} \ \dots \ \gamma_{im\mu} \,\right]^T \tag{5.8.7}$$

is given by the usual LS formula

$$\theta_i^\circ = \left(H^T H\right)^{-1} H^T y_i^\circ. \tag{5.8.8}$$

MA models are used more as filters than to model dynamic processes. A particularly important role is played by MA models as parts (noise filters) of more complex models like ARMA(X) ones. In this context we will consider multivariable MA models of the type

$$y_i(t+\nu_i) = w_i(t+\nu_i) + \sum_{j=1}^{m} \sum_{k=1}^{\nu_i} \gamma_{ijk}\, w_j(t+k-1) \quad (i = 1, \dots, m) \tag{5.8.9}$$

that can be written in the equivalent polynomial form

$$D(z)\, y(t) = R(z)\, w(t) \tag{5.8.10}$$

where

$$r_{ii}(z) = z_i^{\nu} + \gamma_{ii\nu_i}\, z^{\nu_i - 1} + \dots + \gamma_{ii2}\, z + \gamma_{ii1} \tag{5.8.11a}$$

$$r_{ij}(z) = \gamma_{ij\nu_i}\, z^{\nu_i - 1} + \dots + \gamma_{ij2}\, z + \gamma_{ij1}. \tag{5.8.11b}$$

$$D(z) = \mathrm{diag}\left[\, z^{\nu_1} \ \dots \ z^{\nu_m} \,\right]. \tag{5.8.12}$$

These models can be estimated substituting μ with ν_i in (5.8.5) , (5.8.6) , (5.8.7) and using (5.8.8) .

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