ST

System Theory

ST.5 REALIZATION OF MISO INPUT–OUTPUT SEQUENCES

Multi–Input Single–Output (MISO) completely observable systems have a structure intrinsically simpler than Multi–Input Multi–Output (MIMO) ones since it is described by a single integer coincident with the system order $\nu = n$. The set of m input–output difference equations (ST.4.17) reduces to the single equation

$$y(t+n) = \sum_{k=1}^{n} \alpha_k \, y(t+k-1) + \sum_{j=1}^{r} \sum_{k=1}^{n+1} \beta_{jk} \, u_j(t+k-1) \tag{ST.5.1}$$

which allows a simple solution of the problem of obtaining a realization of generic input–output sequences, defined as follows.

Problem ST.5.1 (Minimal realization of MISO systems) – Given generic input–output sequences of a linear discrete–time single–output dynamic system, determine a minimal state space model compatible with the sequences and its initial state.

Solution – Define the Hankel matrix of output samples

$$H_k^L(y) = \begin{bmatrix} y(1) & y(2) & \dots & y(k) \\ y(2) & y(3) & \dots & y(k+1) \\ \vdots & \vdots & & \vdots \\ y(L) & y(L+1) & \dots & y(L+k-1) \end{bmatrix} \tag{ST.5.2}$$

and, in a completely analogous way, the Hankel matrices of input samples $H_k^L(u_1)$, $\dots$, $H_k^L(u_r)$. Define then the composite matrix

$$H_k^L = \left[\, H_k^L(y) \; H_k^L(u_1) \; \dots \; H_k^L(u_r) \,\right] \tag{ST.5.3}$$

and test the rank of the matrices in the sequence

$$H_2^L \; H_3^L \; \dots \; H_k^L \; \dots \tag{ST.5.4}$$

where L is selected large enough to avoid the presence of matrices with less rows than columns. Since the data have been generated, by assumption, by a process that can be described by relation (ST.5.1), we will find maximal rank matrices for $k = 2, 3, \ldots, n$; rank deficient matrices will however be found for $k = n+1, n+2, \ldots$ because of the presence of $1, 2, \ldots$ linear relations between their columns. The first matrix exhibiting a rank deficiency, H_k^L, will thus define the order of the model, $n = k - 1$. The dependence coefficients of the $(n+1)$–th vector of H_{n+1}^L from the first n vectors are, obviously, the parameters α_i while the dependence coefficients from the columns of $H_{n+1}^L(u_1), \ldots, H_{n+1}^L(u_r)$ are the parameters β_{ij}.

The determination of n and of the α_i and β_{ij} parameters allows to write the matrices $\tilde{A}$ and $\tilde{C}$ of the state space model. They also allow to write the matrices M (ST.4.16) and $\overline{B}$ (ST.4.14) that lead, by inverting M, to

$$\Phi = M^{-1}\overline{B} \tag{ST.5.5}$$

whose entries define $\tilde{B}$ and $\tilde{D}$. The initial state associated with the sequences can finally be obtained by means of relation (ST.4.2).

Remark ST.5.1 – The solution of Problem ST.5.1 assumes, implicitly, that the matrix

$$\left[\, H_{n+1}^L(u_1)\ \ldots\ H_{n+1}^L(u_r) \,\right] \tag{ST.5.6}$$

has maximal rank. This is a necessary and sufficient realizability condition that can be easily tested at every step of the procedure.

Remark ST.5.2 – If it is known that the data have been generated by a purely dynamic systems, the matrix H_k^L (ST.5.3) can be substituted by

$$H_k^L = \left[\, H_k^L(y)\ H_{k-1}^L(u_1)\ \ldots\ H_{k-1}^L(u_r) \,\right]. \tag{ST.5.7}$$