ID9

ARAR(X) Identification

9.3 ESTIMATION OF ARARX MODELS

The parameters of $q(z^{-1})$ and $p(z^{-1})$ can be estimated using the instrumental variable method. It is particularly advantageous to use instruments generated by means of an ARX model with order n^* estimated from the ARARX input–output sequences by means of least squares, that give a consistent estimate of the coefficients of $q^*(z^{-1}) = s(z^{-1})q(z^{-1})$ and $p^*(z^{-1}) = s(z^{-1})\,p(z^{-1})$; the instruments

$$y_s^*(t) = \frac{\hat{p}^*(z^{-1})}{\hat{q}^*(z^{-1})}\,u(t) \simeq \frac{p(z^{-1})}{q(z^{-1})}\,u(t) \tag{9.3.1}$$

constitute a good approximation of the ideal instruments $y_s(t) = (p(z^{-1})/q(z^{-1}))u(t)$. After the IV estimate of $p(z^{-1})$ and $q(z^{-1})$ it is possible to compute the equation error

$$e(t) = q(z^{-1})\,y(t) - p(z^{-1})\,u(t) \tag{9.3.2}$$

which allows a consistent least squares estimate of the AR noise model

$$e(t) = \frac{1}{s(z^{-1})}\,w(t) \tag{9.3.3}$$

that completes the ARARX model. The identification procedure can be continued computing the covariance matrices (6.7.7) of the IV estimate of the α_i and β_i parameters and (3.8.5) of the least squares estimate of the δ_i parameters. The validation of the model can be performed analyzing the whiteness of the residuals

$$\varepsilon(t) = y(t) - y(t|t-1) = s(z^{-1})\,q(z^{-1})\,y(t) - s(z^{-1})\,p(z^{-1})\,u(t) \tag{9.3.4}$$

and their correlation with input samples. The whole procedure that has been outlined leads to an asymptotically unbiased estimate of the model parameters that can be obtained analytically; the parameters are, however, obtained applying two different algorithms and the first estimate conditions, through the computation of $e(t)$, the second one. A possible alternative consists in considering the process as ARX and in limiting the estimate to $q(z^{-1})^*$ and $p(z^{-1})^*$. This choice is fully compatible with the use of the model for predictive and control applications but does not allow any separation of the deterministic and stochastic parts of the model because of the inevitable approximations that prevent recognizing the presence of a common factor in $q(z^{-1})^*$ and $p(z^{-1})^*$; it is thus unsuitable for other applications like for instance, fault diagnosis.

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