ID5

MA Identification

5.4 INVERSE YULE–WALKER EQUATIONS

Consider the covariance function of the MA process (5.1.1)

$$r_y(k) = \mathrm{E}\left[\big(y(t+k) - m_y\big)\big(y(t) - m_y\big)\right] \tag{5.4.1}$$

where $m_y = \mathrm{E}[y(t)]$. The spectral density of the process is defined as

$$\Phi(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} r_y(k)\, e^{jk\omega}. \tag{5.4.2}$$

Introduce now the following quantities

$$\rho_k = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \frac{1}{\Phi(\omega)}\, e^{jk\omega}\, \mathrm{d}\omega; \tag{5.4.3}$$

it is possible to verify that the coefficients of the MA process satisfy the “inverse” Yule–Walker equations

$$\rho_k + \gamma_n\, \rho_{k-1} + \ldots + \gamma_1 \rho_{k-n} = 0, \qquad k \geq n. \tag{5.4.4}$$

These equations can be used to compute the parameters of the model when the quantities ρ_i are known. The ρ_i could be computed estimating $\Phi(\omega)$ or also using the AR approximation of the MA process which leads to the following estimates

$$\hat{\rho}_i = \sum_{k=1}^{n_{AR}-i+1} \alpha_k\, \alpha_{k+i}, \tag{5.4.5}$$

where $\alpha_{n_{AR}+1} = -1$. Writing then the inverse Yule–Walker equations for $k = n, n+1, \ldots, 2n-1$, we obtain the following system of equations

$$\begin{bmatrix} \hat{\rho}_{n-1} & \hat{\rho}_{n-2} & \cdots & \hat{\rho}_0 \\ \hat{\rho}_n & \hat{\rho}_{n-1} & \cdots & \hat{\rho}_1 \\ \vdots & \vdots & \ldots & \vdots \\ \hat{\rho}_{2n-2} & \hat{\rho}_{2n-3} & \cdots & \hat{\rho}_{n-1} \end{bmatrix} \begin{bmatrix} \gamma_n \\ \gamma_{n-1} \\ \vdots \\ \gamma_1 \end{bmatrix} = - \begin{bmatrix} \hat{\rho}_n \\ \hat{\rho}_{n+1} \\ \vdots \\ \hat{\rho}_{2n-1} \end{bmatrix} \tag{5.4.6}$$

which allows estimating the parameters of the MA model. System (5.4.6) can also be solved using Levinson–like algorithms.

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