ID6

ARMAX Identification

6.19 PARAMETRIC IDENTIFICATION OF MULTIVARIABLE ARMAX MODELS

Making reference to an assumed structure $\nu = (\nu_1, \ldots, \nu_m)$, it is possible to estimate the parameters of $Q(z)$ and $P(z)$ using, as in the MISO case, instrumental variable methods. The generation of the instruments can be performed using initially even a crude approximation of the system given, for instance, by an ARX model with the assigned structure. This first model can be used to generate a first set of instruments $\eta(t)$ which allows to obtain a first IV estimate of $Q(z)$ and $P(z)$ used to generate new instruments and so on until convergence is achieved. No significant variations separate this step from the MISO case; it is sufficient to redefine y° and θ° according to (3.18.2) and (3.18.3) and to substitute Z with

$$\begin{aligned} Z_i &= Z(\nu_{i1}, \ldots, \nu_i, \ldots, \nu_{im}) \\ &= \left[\, H_{\nu_{i1}}(\eta_1), \ldots, H_{\nu_i}(\eta_i), \ldots, H_{\nu_{im}}(\eta_m), H_{\nu_i}(u_1), \ldots, H_{\nu_i}(u_r) \,\right] \end{aligned} \tag{6.19.1}$$

to use the IV formula

$$\theta_i^\circ = \left(Z_i^T H_i\right)^{-1} Z_i^T y_i^\circ. \tag{6.19.2}$$

Once that an IV estimate of $Q(z)$ and $P(z)$ has been obtained, it is possible to compute the equation error using the relation

$$e(t) = Q(z)^* y(t) - P(z)^* u(t); \tag{6.19.3}$$

The final step consists in estimating the parameters of the MA part of the process, described by the relation

$$D(z)\, e(t) = \big(Q(z) + R(z)\big) w(t) = S(z)\, w(t) \tag{6.19.4}$$

according to the procedure already described for multivariable MA processes.

Remark 6.19.1 – The estimation of $S(z)$ in (6.19.4) faces the identifiability problems mentioned in ID6.17. Since in this two–step procedure $Q(z)$ and $S(z)$ are estimated in cascade it is possible, once that $w(t)$ has been estimated using an auxiliary high–order AR model, to compute the sequence

$$e^*(t) = e(t) - D(z)^{-1} Q(z)\, w(t) \tag{6.19.5}$$

and estimate the parameters of the MA process

$$D(z)\, e^*(t) = R(z)\, w(t). \tag{6.19.6}$$

Note that the polynomial matrix $R(z)$ describing the MA process (6.19.6) has a reduced parameterization induced by the structure ν of the model as anticipated in ID5.8.