

ID5

MA

Identification

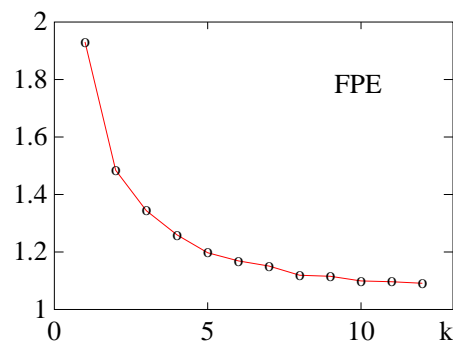
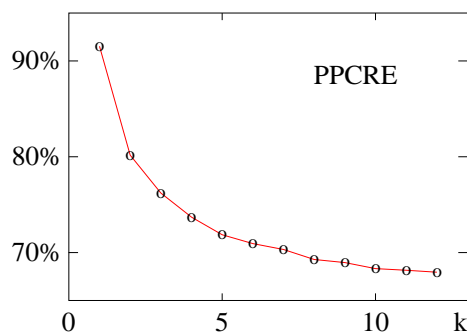


5.3 EXAMPLE 5.1

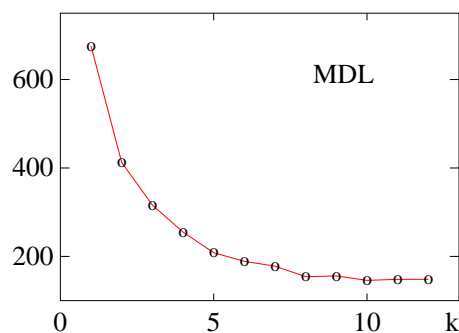
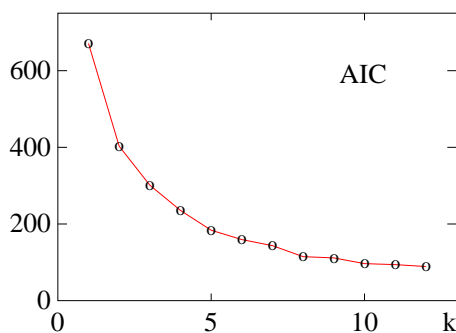
A sequence of 1040 samples has been generated with the MA process

$$y(t) = w(t) + w(t-1) - 0.19w(t-2) - 0.28w(t-3). \quad (5.3.1)$$

Applying PPCRE, FPE, AIC and MDL criteria to estimate an approximating high-order AR model we obtain the results reported in [Figures 5.3.1 – 5.3.4](#), which refer to models with orders up to 12.



Figures 5.3.1 and 5.3.2 – PPCRE and FPE criteria



Figures 5.3.3 and 5.3.4 – AIC and MDL criteria

It can be observed that all criteria show similar behaviors and that none of them allows to determine without uncertainty a specific order; they suggest however, the selection of an order not far from the upper limit of the interval that has been considered. This result is not surprising since, from a theoretical point of view, the order of the auxiliary AR model is infinite. A more sound evaluation can be performed testing the whiteness of the model residuals; the plot of $\zeta_{1020,8}$ reported in Figure 5.3.5, where the dashed line refers to a 99% confidence level for $\chi^2(8)$, shows that an AR model with order 10 allows a good margin.

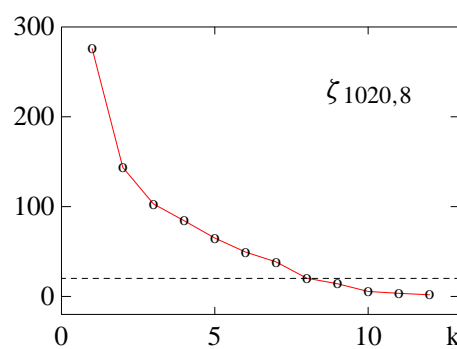
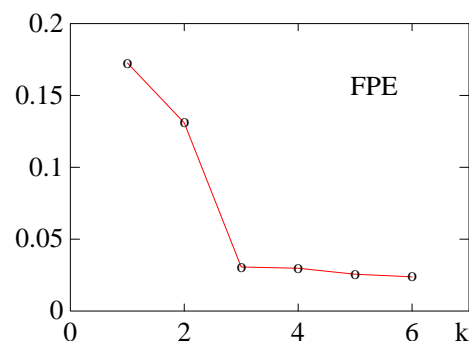
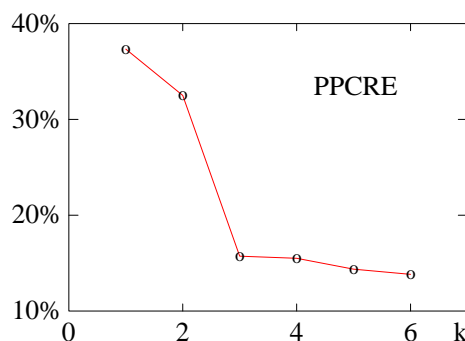


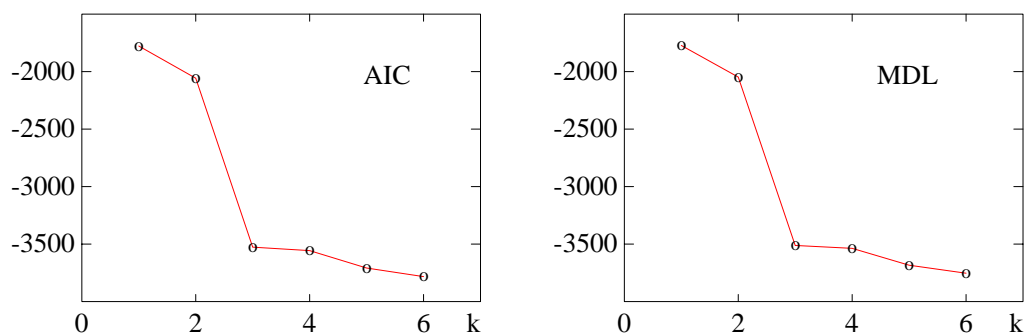
Figure 5.3.5 – $\zeta_{1020,8}$ for AR models with order 1–12

An order 10 AR model has thus been estimated using least squares and its residuals have been considered as remote input, $w(t)$, of the MA process.

It is now necessary to estimate the order of the MA process; this step has been performed with the PPCRE, FPE, AIC and MDL criteria making reference to expression (5.2.6) for y° . The values obtained, taking $N = 1000$, for orders between 1 and 6 are reported in Figures 5.3.6 – 5.3.9. As happened in the evaluation of the order for the auxiliary AR model, also now the different criteria exhibit similar behaviors and do not give unambiguous indications; all show however, sharp changes passing from order 3 to 4 and this suggests to assume 3 as correct order for the model.



Figures 5.3.6 and 5.3.7 – PPCRE and FPE criteria for MA models ($N = 1000$)



Figures 5.3.8 and 5.3.9 – AIC and MDL criteria for MA models ($N = 1000$)

The least squares estimate gives the following values

$$\gamma_1 = -0.3037 \text{ } (-0.28)$$

$$\gamma_2 = -0.1944 \text{ } (-0.19)$$

$$\gamma_3 = 0.9875 \text{ } (1)$$

which constitute, thanks also to the length of the sequence, a good approximation of true values (reported in parentheses). [Figure 5.3.10](#) compares the first 300 observed samples with the output reconstruction (black line) obtained with the identified MA model (using the residuals of the auxiliary AR model as estimate of $w(t)$).

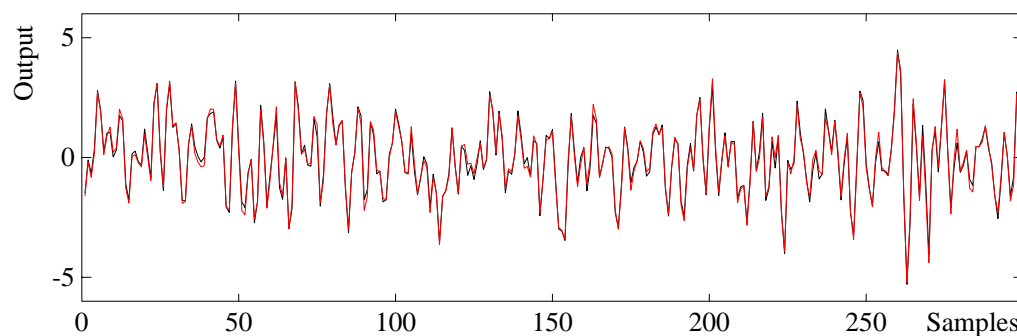


Figure 5.3.10 – Output reconstruction with the identified MA model and observed values

The mean square reconstruction error of this model is $\sigma_\varepsilon^2 = 0.030$ and constitutes a measure of the approximation of the MA process by the auxiliary AR model.

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