ID6

ARMAX Identification

6.5 EXAMPLE 6.1

This example concerns sequences of 520 input–output samples of the ARMAX process generated with the model

$$y(t) = 0.6\,y(t-1) - 0.34\,y(t-2) + 0.4564\,u(t-1) + 0.2738\,u(t-2) \\ + w(t) - 1.1\,w(t-1) + 0.3\,w(t-2). \qquad (6.5.1)$$

The α_i and β_i parameters are the same as in Example 3.1 and also the input sequence is the same, with null mean value and unitary variance, plotted in Figure 3.15.1 . The remote white process $w(t)$ is Gaussian and the variance of the colored noise $v(t) = F(z)w(t)$ is $\sigma_v^2 = 0.0549$; the variance of $y(t)^* = G(z)u(t)$ is, on the whole sequence, $\sigma_{y^*}^2 = 1.0018$. With reference to the decomposition illustrated in Figure 6.1.2 , we could interpret previous figures as the presence of an amount of noise equal to

$$100\,\frac{\sigma_v}{\sigma_{y^*}} = 23.4\%$$

on the data. Using as instruments past input values, according to expression (6.3.10), we obtain, for $n = 2$ and $N = 500$, the following estimate

$$\alpha_1 = -0.3029\ (-0.34) \qquad \beta_1 = 0.3427\ (0.2738)$$
$$\alpha_2 = 0.4998\ (0.6) \qquad \beta_2 = 0.4583\ (0.4564).$$

Generating the instruments with a model of the process it is necessary to start from a first estimate of its parameters that could be, for instance, that obtained using as instruments past inputs, or the least squares estimate, given, in our case, by

$$\alpha_1 = -0.1531\ (-0.34) \qquad \beta_1 = 0.5806\ (0.2738)$$
$$\alpha_2 = 0.1189\ (0.6) \qquad \beta_2 = 0.4741\ (0.4564).$$

Using this estimate to generate the instruments $\eta(t)$ and constructing the matrix of instruments (6.3.7) , we obtain the IV estimate

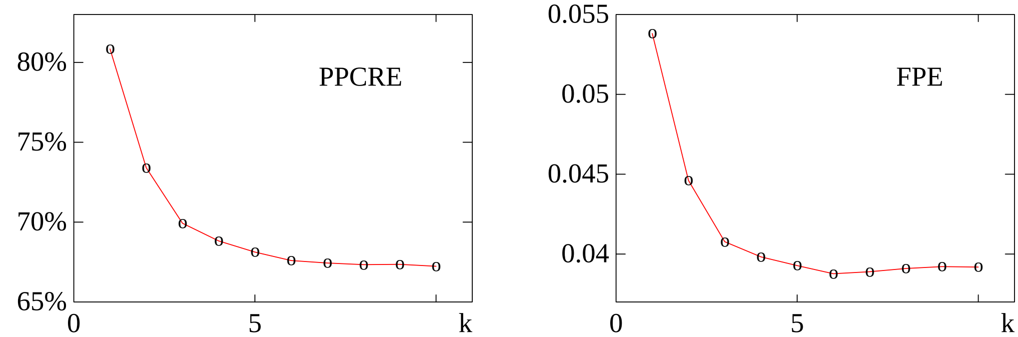
$$\begin{aligned} \alpha_1 &= -0.4603\ (-0.34) & \qquad \beta_1 &= -0.1340\ (0.2738) \\ \alpha_2 &= 0.1943\ (0.6) & \qquad \beta_2 &= 0.9118\ (0.4564) \end{aligned}$$

that allows constructing new instruments and a new IV estimate. Iterating this procedure we achieve, in 3 steps, a convergence to the final estimate

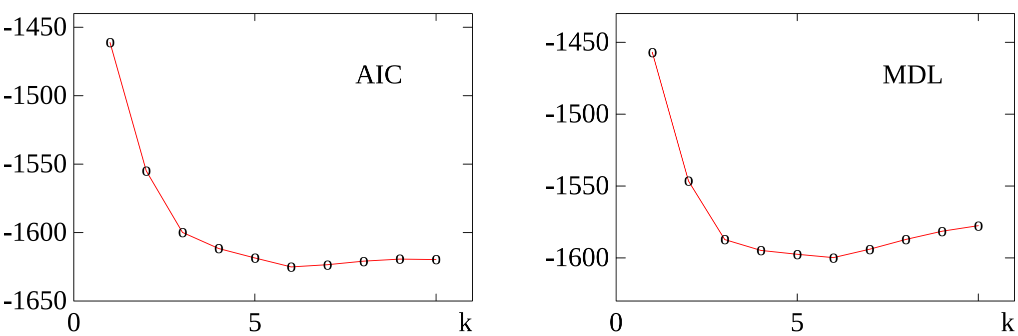
$$\begin{aligned} \alpha_1 &= -0.3224\ (-0.34) & \qquad \beta_1 &= 0.3253\ (0.2738) \\ \alpha_2 &= 0.5375\ (0.6) & \qquad \beta_2 &= 0.4535\ (0.4564). \end{aligned}$$

The choice of the initial estimate does not influence the final values but only the number of steps that are necessary for convergence.

It is now possible to compute the equation error by means of (6.4.1) and to model this sequence by means of a MA process. The first step requires estimating a high order auxiliary AR model to approximate the sequence; the order determination test for AR models with orders in the range 1–10 are reported in Figures 6.5.1 – 6.5.4 .



Figures 6.5.1 and 6.5.2 – PPCRE and FPE criteria



Figures 6.5.3 and 6.5.4 – AIC and MDL criteria

Performing a whiteness test on the residuals of the AR models by computing $\zeta_{500,8}$ we obtain the results reported in Figure 6.5.5 where the dashed line refers to a confidence level of 99%; it can be observed that and order 6 model fully satisfies the whiteness requirement.

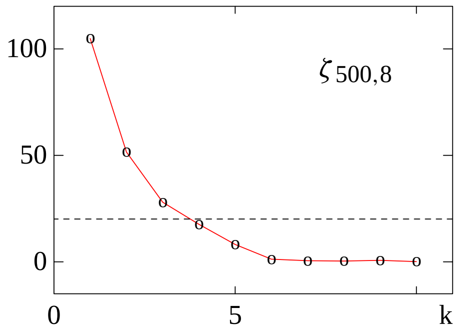


Figure 6.5.5 – $\zeta_{500,8}$ for AR models with order 1–10

Estimating the parameters of an order 6 AR model with least squares we obtain

$$
\begin{aligned}
\alpha_1 &= -0.1260 \\
\alpha_2 &= -0.2688 \\
\alpha_3 &= -0.4281 \\
\alpha_4 &= -0.6564 \\
\alpha_5 &= -0.9117 \\
\alpha_6 &= -1.0732.
\end{aligned}
$$

The subsequent step consists in computing the residuals of the AR model assuming them as estimate of the remote white process $w(\cdot)$ that can finally be used to estimate the γ_i coefficients describing the MA part of the process. Using (5.2.4)–(5.2.6) we obtain

$$
\begin{aligned}
\gamma_1 &= \quad 0.2402 \ (0.3) \\
\gamma_2 &= -1.0723 \ (-1.1).
\end{aligned}
$$

Considering an order 8 auxiliary AR model we would obtain

$$
\begin{aligned}
\gamma_1 &= \quad 0.2432 \ (0.3) \\
\gamma_2 &= -1.0824 \ (-1.1),
\end{aligned}
$$

i.e. slightly better values. Now that all parameters have been estimated, it is possible to implement predictor (6.2.2) and validate the ARMAX model testing the whiteness of its residuals. Figure 6.5.6 reports the one–step–ahead model prediction (black line) and the observed output (MA parameters are those obtained with an order 6 auxiliary AR model). The residuals are reported in Figure 6.5.7 .

The validation has been performed computing $\zeta_{500,8}$ for the sequence of residuals, given by

$$\zeta_{500,8} = 5.1685;$$

this value, well below the limit corresponding to a 99% confidence level for $\chi^2(8)$, assures that the model gives a good description of the considered ARMAX process.

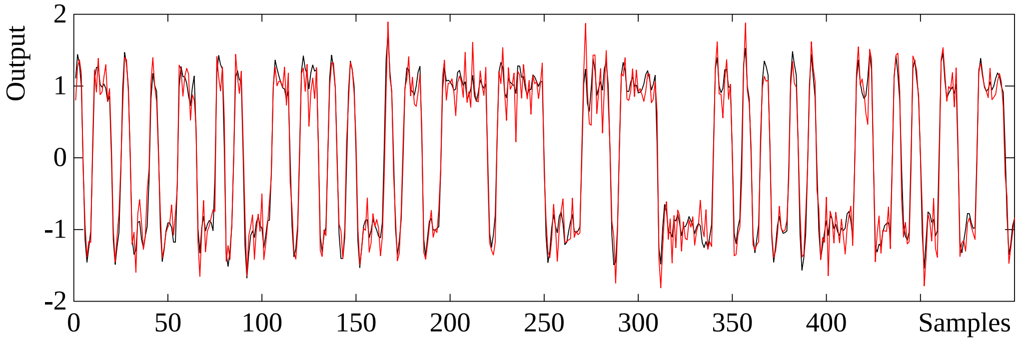


Figure 6.5.6 – Model prevision (black line) and observed output

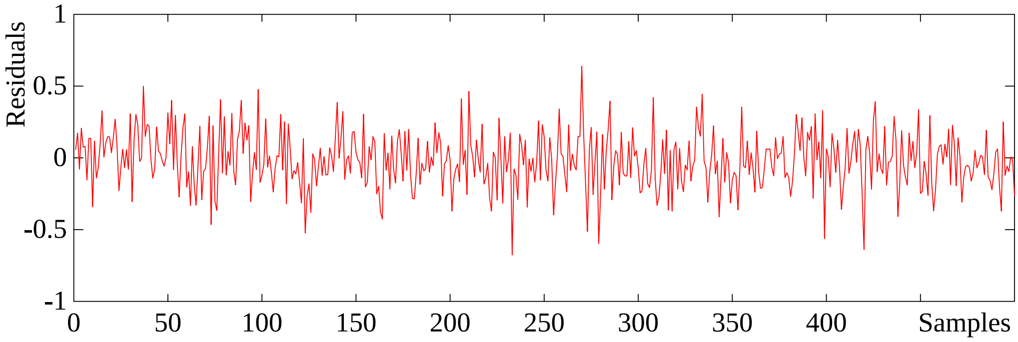


Figure 6.5.7 – Residuals of the ARMAX model

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