ID3

ARX Identification

3.8 COVARIANCE OF LEAST SQUARES ESTIMATES

The covariance matrix of the estimate θ° gives indications on the dispersion of the values obtained and on the correlations between the estimates of single parameters that can be useful both in designing and evaluating identification experiments. It is given by

$$\operatorname{cov}\theta^\circ = \mathrm{E}\left[(\theta^\circ - \mathrm{E}[\theta^\circ])(\theta^\circ - \mathrm{E}[\theta^\circ])^T\right], \tag{3.8.1}$$

and is usually written in the form

$$\operatorname{cov}\theta^\circ = \mathrm{E}\left[(\theta^\circ - \theta^*)(\theta^\circ - \theta^*)^T\right], \tag{3.8.2}$$

i.e. assuming that $\mathrm{E}[\theta^\circ] = \theta^*$; relation (3.8.2) can be rewritten in the form

$$\begin{aligned} \operatorname{cov}\theta^\circ &= \mathrm{E}\left[\left((H^TH)^{-1}H^Ty^\circ - \theta^*\right)\left((H^TH)^{-1}H^Ty^\circ - \theta^*\right)^T\right] \\ &= \mathrm{E}\left[(H^TH)^{-1}H^Te^\circ e^{\circ T}H(H^TH)^{-1}\right] \end{aligned} \tag{3.8.3}$$

which, because of the whiteness of $e(\cdot)$ and of the time lags between the entries of $e(\cdot)$ and those of H, is equivalent to expression

$$\operatorname{cov}\theta^\circ = \mathrm{E}\left[(H^TH)^{-1}H^T \operatorname{cov}e^\circ\, H(H^TH)^{-1}\right] = \sigma_e^2\, \mathrm{E}\left[(H^TH)^{-1}\right] \tag{3.8.4}$$

where σ_e^2 denotes the variance of process $e(\cdot)$. The expected value operator could be omitted in (3.8.4) only under the assumption of deterministic entries in H; even if this is not true, the commonly used approximation is

$$\operatorname{cov}\theta^\circ = \sigma_e^2\left(H^TH\right)^{-1} \tag{3.8.5}$$

easily implemented because of the availability, in least squares estimation, of the matrix $(H^TH)^{-1}$. Using weighted least squares, relation (3.8.4) becomes

$$\operatorname{cov}\theta^\circ = \sigma_e^2\, \mathrm{E}\left[(H^TWH)^{-1}H^TWW^TH(H^TWH)^{-1}\right]. \tag{3.8.6}$$

It will be shown, in the following, that the covariance of least squares estimates (3.8.4) is minimal over the class of all non biased estimates θ^*.