



LA

Linear Algebra



LA.5 PROPERTIES OF PARTITIONED MATRICES

Lemma LA.5.1 – Consider the partitioned matrix

$$M = \begin{bmatrix} A + BCD \end{bmatrix}; \quad (\text{LA.5.1})$$

if both A and C are invertible, then

$$M^{-1} = A^{-1} - A^{-1}B \begin{bmatrix} C^{-1} + DA^{-1}B \end{bmatrix}^{-1} DA^{-1}. \quad (\text{LA.5.2})$$

Lemma LA.5.2 – Consider the partitioned matrix

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (\text{LA.5.3})$$

where A and D are square matrices. If A and $(D - CA^{-1}B)$ are both nonsingular, then

$$M^{-1} = \begin{bmatrix} A^{-1} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -A^{-1}B \\ I \end{bmatrix} \begin{bmatrix} D - CA^{-1}B \end{bmatrix}^{-1} \begin{bmatrix} -CA^{-1} & I \end{bmatrix}. \quad (\text{LA.5.4})$$

Similarly, if D and $(A - BD^{-1}C)$ are nonsingular, then

$$M^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & C^{-1} \end{bmatrix} + \begin{bmatrix} I \\ -D^{-1}C \end{bmatrix} \begin{bmatrix} A - BD^{-1}C \end{bmatrix}^{-1} \begin{bmatrix} I & -BD^{-1} \end{bmatrix}. \quad (\text{LA.5.5})$$

Lemma LA.5.3 – Consider the partitioned matrix M (LA.5.3). If A is nonsingular then

$$\det M = \det A \det (D - CA^{-1}B) \quad (\text{LA.5.6})$$

and, if D is nonsingular then

$$\det M = \det D \det (A - BD^{-1}C). \quad (\text{LA.5.7})$$

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