ID6

ARMAX Identification

6.14 EXAMPLE 6.4

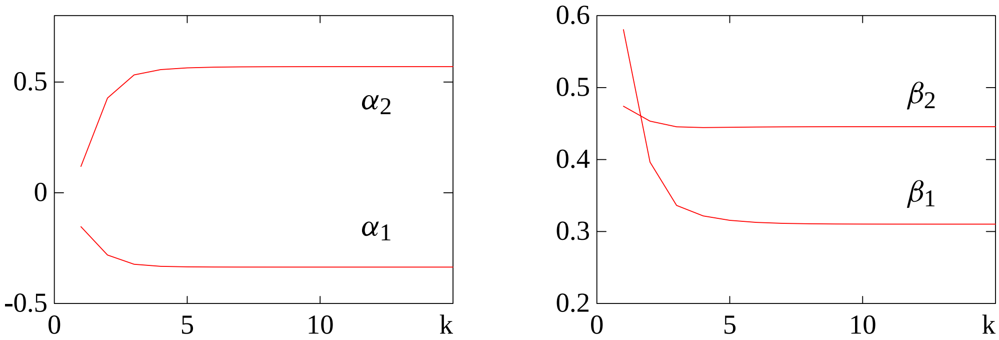
The ARMAX process described in Example 6.1 will be considered again to perform a PEM estimation of its parameters using the Gauss–Newton algorithm. We will start from a poor initial estimate of the parameters to show the good convergence properties of the algorithm; for this purpose the α_i and β_i parameters are those obtained applying least squares and the γ_i parameters are those of the MA process that can be estimated from the equation error of the least squares model, given by

$$\gamma_1 = -0.1751 \ (0.3)$$
$$\gamma_2 = -0.5240 \ (-1.1);$$

also, these parameters constitute a poor estimate of real ones, Predictor (6.2.2) has then been used to obtain the sequence of prediction errors $\varepsilon(t, \theta^0)$ associated with the initial parameter estimate

$$\theta^0 = \begin{bmatrix} -0.1531 & 0.1189 & 0.5806 & 0.4741 & -0.1751 & -0.5240 \end{bmatrix}^T.$$

Filtering the sequences $y(\cdot)$, $u(\cdot)$ and $\varepsilon(\cdot, \theta^0)$ with the filter $1/r(z^{-1})$ we obtain the sequences $y^F(\cdot)$, $u^F(\cdot)$ and $\varepsilon^F(\cdot, \theta^0)$ that allow the construction of matrix H_ψ (6.13.17) and finally, the application of (6.13.19) to obtain the new estimate θ^1.



Figures 6.14.1 and 6.14.2 – Evolution of the estimates of coefficients α_i and β_i

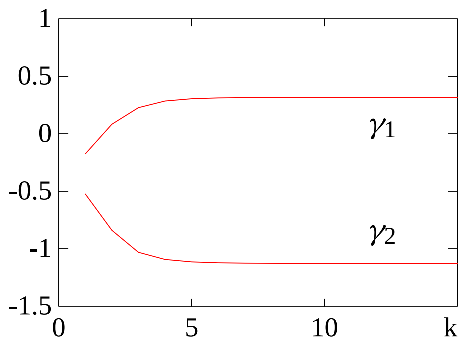


Figure 6.14.3 – Evolution of the estimates of coefficients γ_i

The procedure is then iterated until convergence; the value of Δ_k in (6.13.19) has been kept constant (equal to $1/\sqrt{2}$) in all steps. The values of the parameters α_i, β_i and γ_i in the different steps are reported in Figures 6.14.1 – 6.14.3 where it is possible to observe a fast convergence rate; after few iterations the parameter values, despite the bad initial values, approximate well final ones and after ten iterations the fourth digit does not change any more. The final values are

$$\begin{array}{lll} \alpha_1 = -0.3356\ (-0.34) & \beta_1 = \ 0.3103\ (0.2738) & \gamma_1 = \ 0.3163\ (0.3) \\ \alpha_2 = \ 0.5695\ (0.6) & \beta_2 = \ 0.4456\ (0.4564) & \gamma_2 = -1.1272\ (-1.1). \end{array}$$

Figure 6.14.4 reports the one–step–ahead prevision of this model (black line) against observed values; the residuals are reported in Figure 6.14.5. The mean square prevision error of this PEM model is $J^{ML}_{500}(\theta) = 0.0376$, only slightly better than the error of the IV model, given by $J^{IV}_{500}(\theta) = 0.0379$. The whiteness test on the residuals gives

$$\zeta_{500,8} = 2.8989,$$

which confirms the validity of the model.

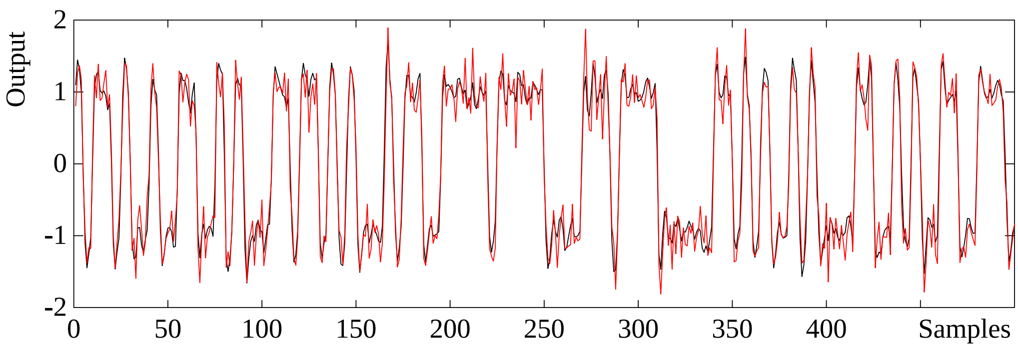


Figure 6.14.4 – PEM model prevision (black line) and observed output

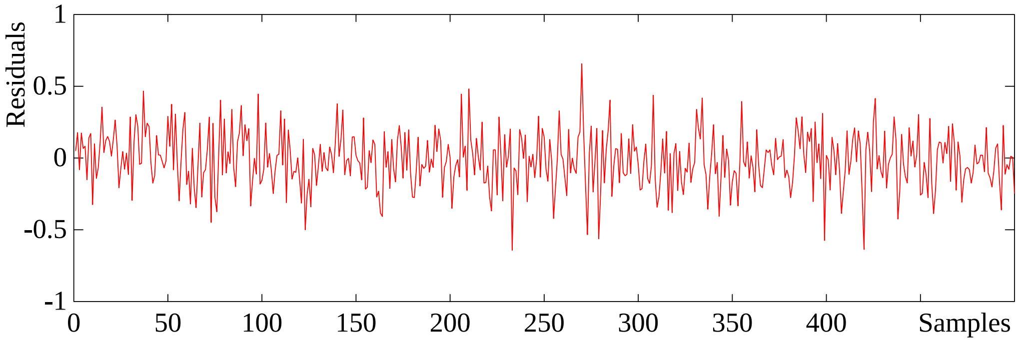


Figure 6.14.5 – Residuals of the PEM model