ST

System Theory

ST.2 EQUIVALENCE RELATIONS FOR MFD MODELS

Definition ST.2.1 (Equivalence of MFD models) – Two MFD models $(Q'(z), P'(z))$ and $(Q''(z), P''(z))$ are defined as equivalent when they define the same transfer function, i.e. when

$$Q'(z)^{-1} P'(z) = Q''(z)^{-1} P''(z). \tag{ST.2.1}$$

Remark ST.2.1 – The definition of equivalence that has been given does not imply that $\deg\det Q'(z) = \deg\det Q''(z)$. Equivalent pairs are linked by the relation

$$Q''(z) = M(z)\, Q'(z) \tag{ST.2.2a}$$
$$P''(z) = M(z)\, P'(z) \tag{ST.2.2b}$$

where $M(z)$ is a rational matrix. Note that limiting $M(z)$ to the class of generic (non unimodular) polynomial matrices we do not obtain an equivalence relation because the inverses of generic polynomial matrices are rational matrices.

Definition ST.2.2 (Strict equivalence of MFD models) – Two MFD models $(Q'(z), P'(z))$ and $(Q''(z), P''(z))$ are defined as strictly equivalent when they define the same transfer function and

$$\det Q''(z) = \alpha \det Q'(z). \tag{ST.2.3}$$

Remark ST.2.2 – The definition of strict equivalence implies an algebraic link between equivalent pairs still given by (ST.2.2) where, however, $M(z)$ is a unimodular matrix. (ST.2.2) constitutes, in this case, an equivalence relation because the inverses of unimodular matrices are still polynomial matrices.

Definition ST.2.3 (Strict equivalence between state space and MFD models) – A state space model (ST.1.1) and a MFD model (ST.1.6) are defined as strictly equivalent when $n = \dim A = \deg\det Q(z)$ and for every initial state x_0 and input sequence $u(\cdot)$ there exist n initial conditions $y_1(t), \ldots, y_1(t+n_1), \ldots, y_m(t+n_m)$ that determine, in model (ST.1.6), the same output sequence as in model (ST.1.1) with initial state x_0.

Remark ST.2.3 – Every MFD model admits a strictly equivalent state space realization.