# ID8

# ARIMA(X) Identification

## 8.2 EXAMPLE 8.1

The data that will be considered in this example is the measured maximal temperature in the city of Bologna over a period of 134 days.

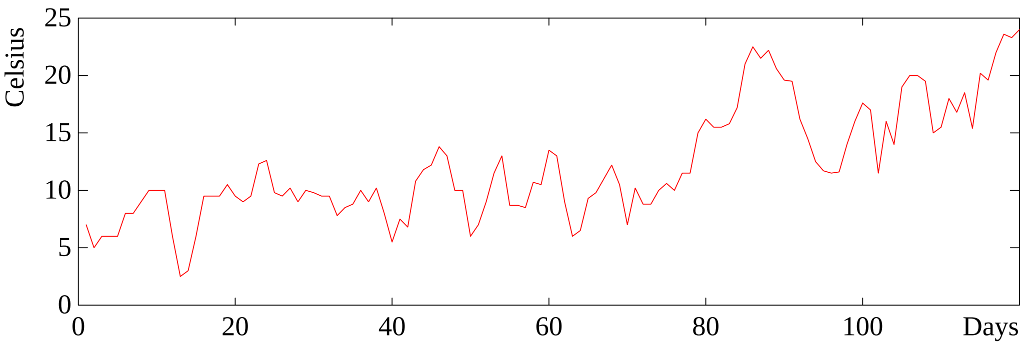
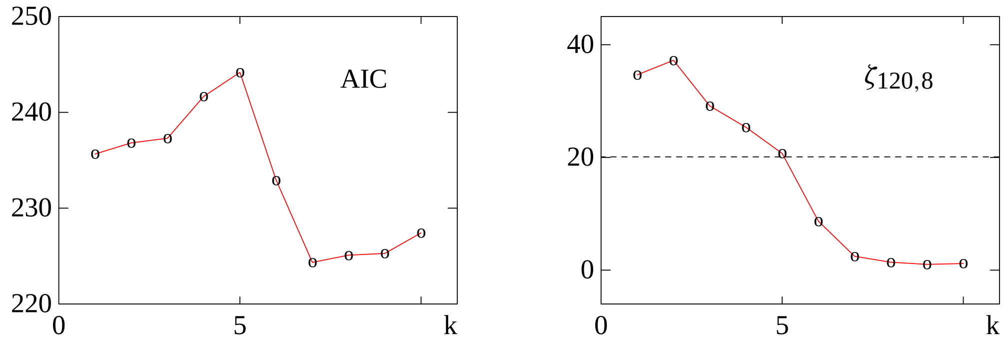



Figure 8.2.1 – Maximal temperature in Bologna over a period of 134 days

These data exhibit (Figure 8.2.1) an evident trend due to the transition from winter to spring and will be described by means of an ARIMA model; the first step consists thus in substituting the data with their differences. Avoiding the description of the order selection procedure and assuming $n = 5$, the second step concerns the estimation of the autoregressive parameters $\alpha_i$, by means of Yule–Walker equations (7.3.6); taking $N = 120$ we obtain

$$\alpha_1 = 0.8526 \qquad \alpha_4 = -0.2575$$
$$\alpha_2 = -1.0701 \qquad \alpha_5 = 0.2583.$$
$$\alpha_3 = 0.3720$$

It is now possible to compute the sequence of equation errors (7.3.8) and model this sequence by means of an auxiliary AR model. PPCRE, FPE, AIC and MDL criteria suggest 7 as most suitable order for this model (see for instance, the values of the AIC criterion in Figure 8.2.2); this choice is confirmed by a whiteness test on its residuals (Figure 8.2.3).

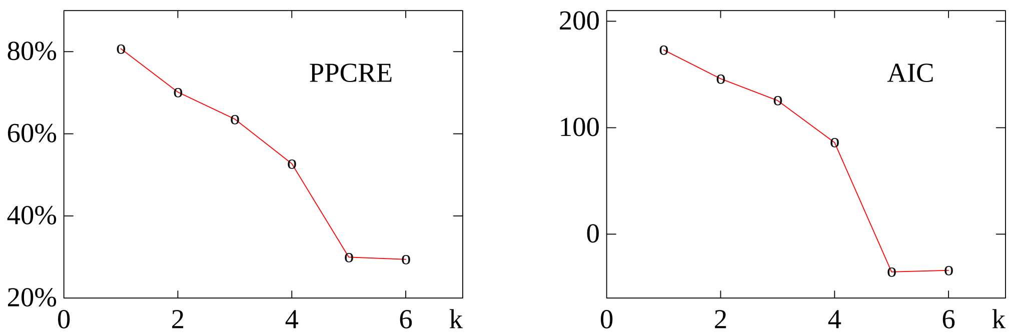



Figures 8.2.2 and 8.2.3 – AIC criterion and whiteness test for auxiliary AR models

The least squares estimate of the parameters is given, for $N = 120$, by

$$
\begin{aligned}
\alpha_1 &= -0.2932 & \qquad \alpha_5 &= -0.0634 \\
\alpha_2 &= -0.4856 & \qquad \alpha_6 &= -0.0323 \\
\alpha_3 &= -0.0954 & \qquad \alpha_7 &= -0.6749. \\
\alpha_4 &= \phantom{-}0.1591 & &
\end{aligned}
$$

The residuals of this model are now assumed as estimate of the remote noise $w(t)$ in order to compute the parameters of the MA part of the model. The order determination criteria indicate 5 as most suitable choice (see the PPCRE and AIC values in Figures 8.2.4 and 8.2.5) and this indication is congruent with the order initially assumed for the AR part of the model. The whiteness test on the residuals shows that this choice is marginal with respect to a confidence level of 99%.




Figures 8.2.4 and 8.2.5 – PPCRE and AIC criteria for the MA part of the model

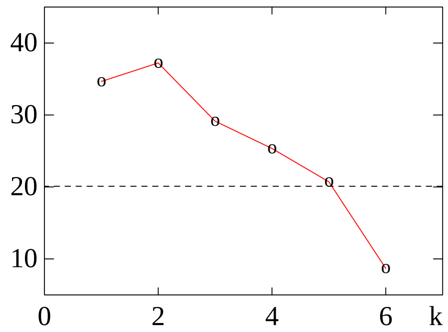


Figure 8.2.6 – Whiteness test on the residuals of the MA model

The least squares estimate of the $\gamma_i$ parameters is

$$\gamma_1 = -0.4859 \qquad \gamma_4 = \phantom{-}0.4057$$
$$\gamma_2 = \phantom{-}0.4325 \qquad \gamma_5 = -0.6675$$
$$\gamma_3 = -0.3374$$

and the mean square prevision error on the first 120 previsions is

$$J^{YW/LS}(\theta) = 5.6056.$$

Using the Gauss–Newton algorithm (6.13.19) with $\Delta_k = 1/\sqrt{2}$ we obtain, after 22 iterations, the following estimate

$$\alpha_1 = \phantom{-}0.7687 \qquad \gamma_1 = -0.7867$$
$$\alpha_2 = -0.6423 \qquad \gamma_2 = \phantom{-}0.4444$$
$$\alpha_3 = -0.2515 \qquad \gamma_3 = \phantom{-}0.1141$$
$$\alpha_4 = \phantom{-}0.1637 \qquad \gamma_4 = -0.0643$$
$$\alpha_5 = \phantom{-}0.2437. \qquad \gamma_5 = -0.3866.$$

The prevision of this PEM model (black line) is compared with the observations in Figure 8.2.7; the residuals are plotted in Figure 8.2.8.

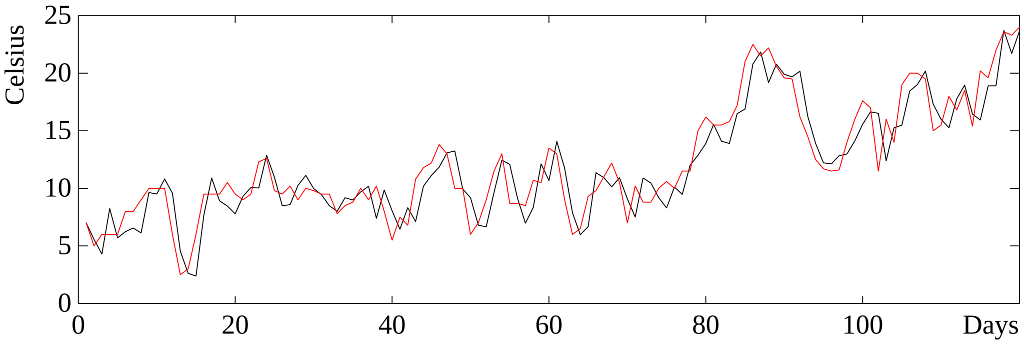


Figure 8.2.7 – Prevision of the PEM ARIMA model

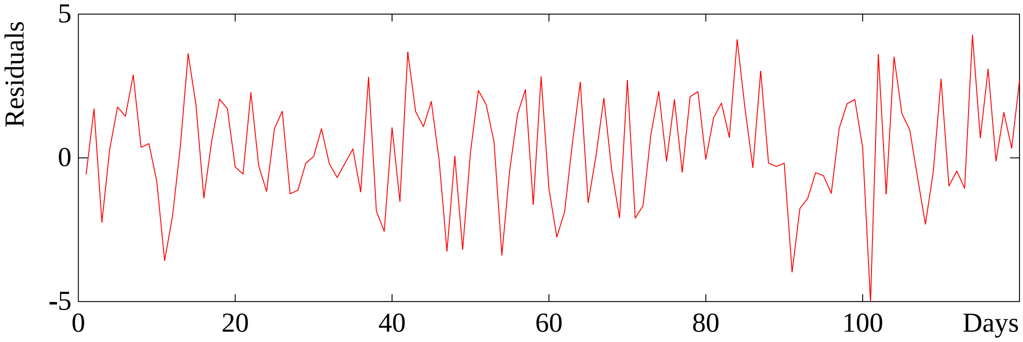



Figure 8.2.8 – Residuals of the PEM ARIMA model

The mean square prediction error associated with this model is

$$J^{ML}(\theta) = 3.5559,$$

remarkably better than the error given by the initial model.

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