Lecture 5.5

Practical Aspects on System Identification and Summary

- Remarks of Persistent Excitation Conditions

Persistent Excitation

Just as for the off-line case, it is important that the input signal changes sufficiently in order to excite the system so that the experimental data contains enough information about the dynamics of the system. This leads to the concept of *persistent excitation* relating to the input signal. Such persistent excitation concept/condition applies during the whole identification period.

Most stable linear systems may be represented by the so called finite impulse response (FIR) model:

$$y(t) = b_1 u(t-1) + b_2 u(t-2) + ... + b_n u(t-n) = \varphi^T(t)\theta$$

where

$$\theta = [b_1 \ \ \ldots \ \ b_n]^T$$

$$\varphi = [u(t-1)\ \ldots\ u(t-n)]^T$$

The estimate is given by $\hat{\theta} = [\Phi^T\Phi]^{-1}\Phi^T Y$, where

$$\Phi^T\Phi = \begin{bmatrix} \sum_{j=n+1}^{t} u(j-1)^2 & \sum_{j=n+1}^{t} u(j-1)u(j-2) & \cdots & \sum_{j=n+1}^{t} u(j-1)u(j-n) \\ \sum_{j=n+1}^{t} u(j-2)u(j-1) & \sum_{j=n+1}^{t} u(j-2)^2 & \cdots & \sum_{j=n+1}^{t} u(j-2)u(j-n) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{j=n+1}^{t} u(j-n)u(j-1) & \sum_{j=n+1}^{t} u(j-n)u(j-2) & \cdots & \sum_{j=n+1}^{t} u(j-n)^2 \end{bmatrix}$$

This matrix has to be non-singular for the estimate to be unique. This is called an excitation condition. For a long data set, $t \rightarrow \infty$, and all sums may be taken from 1 to t.

Define:

$$C_n = \lim_{t\to\infty} \frac{1}{t}\Phi^T\Phi = \begin{bmatrix} c(0) & c(1) & \cdots & c(n-1) \\ c(1) & c(0) & \cdots & c(n-2) \\ \vdots & \vdots & \ddots & \vdots \\ c(n-1) & c(n-2) & \cdots & c(0) \end{bmatrix}$$

where $c(k)$ are the empirical covariances of the input. That is:

$$c(k) = \lim_{t\to\infty} \frac{1}{t}\sum_{j=1}^{t} u(j)u(j-k)$$

Definition: A signal u is called *persistently exciting* (PE) of order n if the matrix C_n is positive definite.

Examples:

- **White noise**: $u(t)$ is white noise, with zero mean and variance σ^2. Then $c(n) = \sigma^2 \delta_n$, and $C_n = \sigma^2 I_n$, which is always positive definite. Thus C_n is nonsingular for all n, and white noise signal is PE of all orders.

- **Step signal**: $u(t)$ is a step of magnitude σ, then $c(k) = \sigma^2$, and C_n is nonsingular only if $n = 1$. Then a step is PE of order 1.

- **Impulse signal**: $u(t) = 1$ for $t = 0$, and 0 otherwise. This gives $c(n) = 0$ for all n and $C_n = 0$. Therefore, this signal is not PE of any order.

Important Note: It is necessary for consistent estimation of an n-th order system that the input signal be at least persistently exciting of order $2n$.