SYSTEM IDENTIFICATION AND DATA ANALYSIS

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General Course Information

- **Lectures:** Monday, 8:30-11:30, lecture room 5;
  Wednesday, 8:30-10:30, lecture room 7;
  Thursday, 8:30-10:30, lecture room 7 (or Lab.).
- **Instructor:** *Silvio Simani*
- **Textbook:**
  Lennart Ljung, *System Identification: Theory for the User*, 2nd Edition, Prentice-Hall, 1999 (Book's web page: http://www.control.isy.liu.se/~ljung/sysid)
- **Reference books:**
  1. L. Ljung and T. Glad, Modeling of Dynamic Systems, Prentice Hall, 1994
  2. T. Soderstrom and P. Stoica, System Identification, Prentice Hall International (UK) Ltd, 1989
- **Course web-page:**
  www.ing.unife.it/simani/lessons.html

Course Outline

1. Introduction and overview on system identification
2. Non-recursive (off-line) identification methods
3. Non-recursive and recursive (on-line) identification methods
4. Recursive identification methods
5. Practical aspects and applications of system identification

Associated Reading in the Textbook

1. Introduction and overview on system identification (Ch. 1; 4.1-4.3; Ch. 6)
2. Non-recursive (off-line) identification methods (Ch. 7)
3. Non-recursive and recursive (on-line) identification methods (Ch. 10; Ch. 11)
4. Recursive identification methods (Ch. 11)
5. Practical aspects and applications of system identification (Ch. 13, 14, 16, 17)

System Identification and Data Analysis

Lecture 1

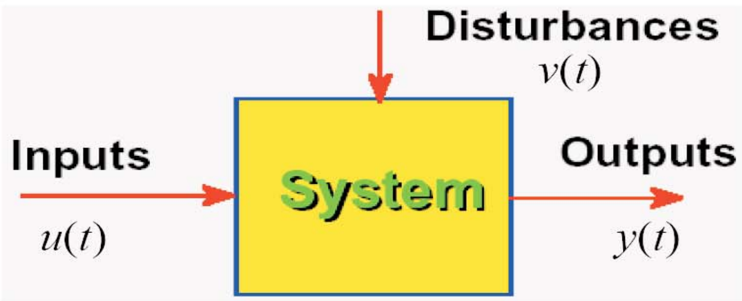
Introduction and Overview

- What is System Identification (SI)?
- Introduction to systems and models
- Procedure of system identification
- Methods of system identification
- Review on topics covered in course "Automatica I (Laboratorio)"
- Examples of system identification

System Identification

"Identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent."

- L. Zadeh, (1962)



System identification is the field of *modeling* dynamic systems from *experimental data*

Systems

System: A collection of components which are coordinated together to perform a function.

A system is a defined part of the real world. Interactions with the environment are described by inputs, outputs, and disturbances.

Dynamic system: A system with a memory, i.e., the input value at time *t* will influence the output at future instants.

Examples of dynamic system: (pp. 2-6, textbook)

- Example 1.1 A Solar-Heated House
- Example 1.2 A Military Aircraft
- Example 1.3 Speech

Ex. A Solar Heated House

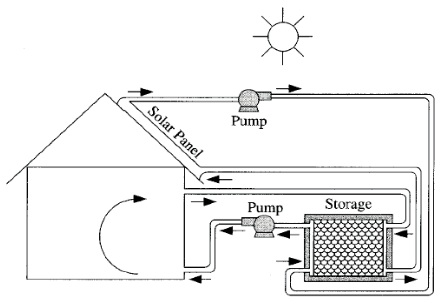


Figure 1.2 A solar-heated house.

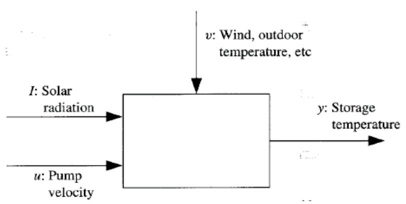


Figure 1.3 The solar-heated house system: u: input; I: measured disturbance; y: output; v: unmeasured disturbances.

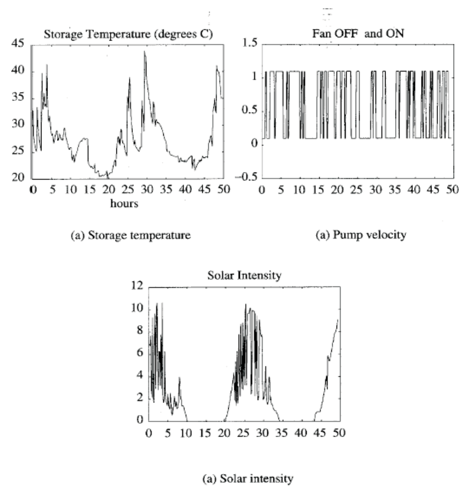


Figure 1.4 Storage temperature y, pump velocity u, and solar intensity I over a 50-hour period. Sampling interval: 10 minutes.

Ex. Speech

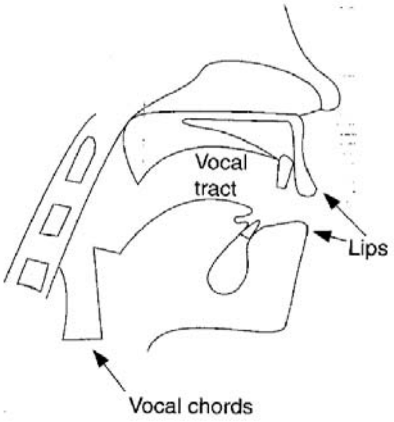


Figure 1.7 Speech generation.

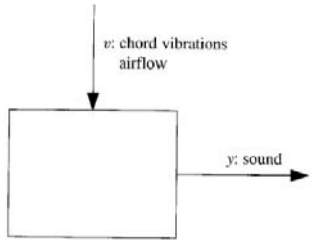


Figure 1.8 The speech system: y: output; v: unmeasured disturbance.

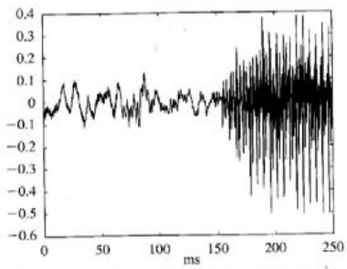
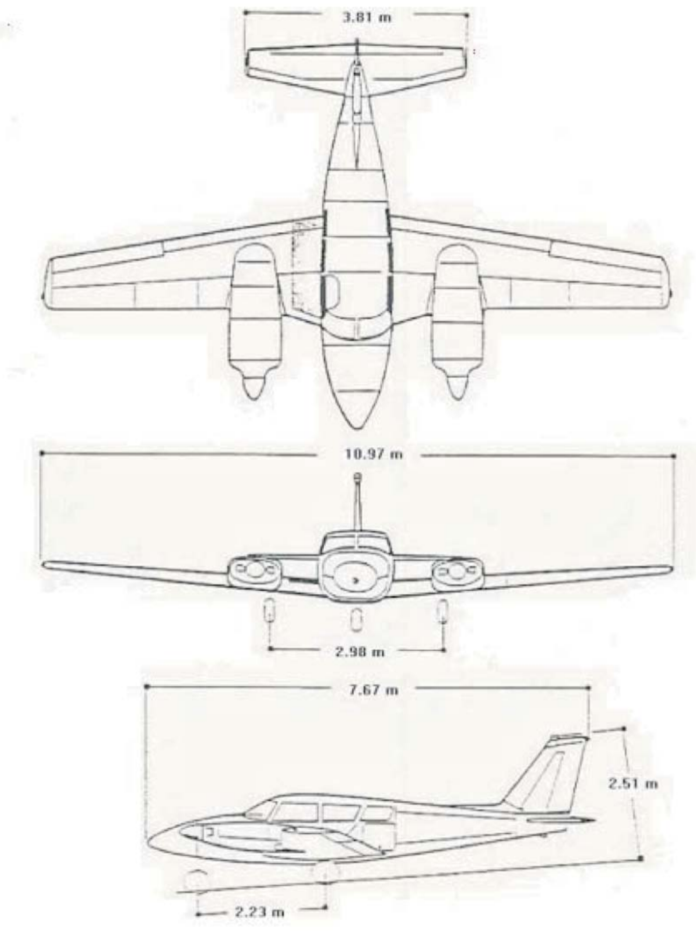


Figure 1.9 The speech signal (air pressure). Data sampled every 0.125 ms. (8 kHz sampling rate).

Aircraft Model

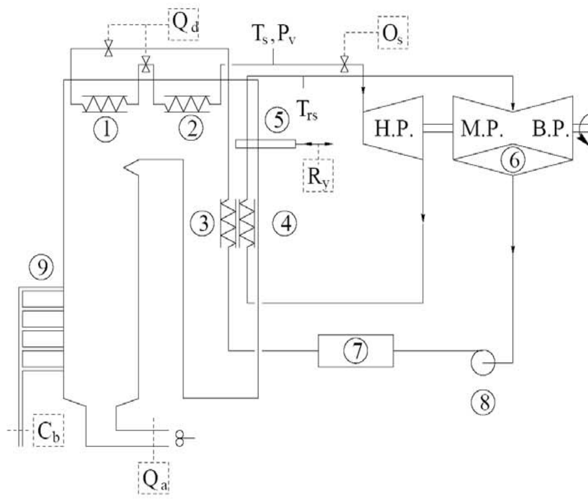


Symbol	Sensor Variable
δ_e	Elevator deflection angle
δ_a	Aileron deflection angle
δ_a	Rudder deflection angle
δ_{th}	Throttle aperture %
V	True Air Speed
Q	Pitch Rate
θ	Elevation Angle
H	Altitude
P	Roll Rate
R	Yaw Rate
ϕ	Bank Angle
ψ	Heading Angle
n	Engine Angular Rate

120 MW Power Plant "Pont sur Sambre"

Process Description

3 major components: the *reactor*, *turbine*, & *condenser*



$u_1(t)$:	C_b	gas flow
$u_2(t)$:	O_s	turbine valves opening
$u_3(t)$:	Q_d	super heater spray flow
$u_4(t)$:	R_y	gas dampers
$u_5(t)$:	Q_a	air flow
$y_1(t)$:	P_v	steam pressure
$y_2(t)$:	T_s	main steam temperature
$y_3(t)$:	T_{rs}	reheat steam temperature

Aircraft Mathematical Model

$$\dot{V} = F_x \frac{\cos\alpha\cos\beta}{m} + F_y \frac{\sin\beta}{m} + F_z \frac{\sin\alpha\cos\beta}{m}$$

$$\dot{\alpha} = \frac{-F_x \sin\alpha + F_z \cos\alpha}{mV\cos\beta} + Q - (P\cos\alpha + R\sin\alpha)\tan\beta$$

$$\dot{\beta} = \frac{-F_x\cos\alpha\sin\beta + F_y\cos\beta - F_z\sin\alpha\sin\beta}{mV} + P\sin\alpha - R\cos\alpha$$

$$\dot{P} = \frac{M_x I_z + M_z I_{xz} + PQ\,I_{xz}(I_x - I_y + I_z)}{I_x I_z - I_{xz}^2} + \frac{QR\left(I_y I_z - I_{xz}^2 - I_z^2\right)}{I_x I_z - I_{xz}^2}$$

$$\dot{Q} = \frac{M_y + PR(I_z - I_x) - P^2 I_{xz} + R^2 I_{xz}}{I_y}$$

$$\dot{R} = \frac{M_x I_{xz} + M_z I_x + PQ\left(I_x^2 - I_x I_y + I_{xz}^2\right)}{I_x I_z - I_{xz}^2} + \frac{QR\,I_{xz}(-I_x + I_y - I_z)}{I_x I_z - I_{xz}^2}$$

$$\dot{\phi} = P + Q\sin\phi\tan\theta + R\cos\phi\tan\theta$$

$$\dot{\theta} = Q\cos\phi - R\sin\phi$$

$$\dot{\psi} = \frac{Q\sin\phi + R\cos\phi}{\cos\theta}$$

$$\dot{H} = V\cos\alpha\cos\beta\sin\theta - V\cos\theta\left(\sin\beta\sin\phi + \sin\alpha\cos\beta\cos\phi\right) - V_{Az}$$

Models

Model: A description of the system. The model should capture the essential information about the system.

Systems	**Models**
Complex	Approximative (However, model should capture the relevant information of the system)
Building/Examining systems is expensive, dangerous, time consuming, etc.	Models can answer many questions about the system.

Types of Models

- Mental, intuitive or verbal models
  - e.g., driving a car
- Graphs and tables
  - e.g., Bode plots and step responses
- Mathematical models
  - e.g., differential and difference equations, which are well-suited for modeling dynamic systems

Mathematical Models and Benifits

- Do not require a physical system
  - ➢ Can treat new designs/technologies without prototype
  - ➢ Do not disturb operation of existing system
- Easier to work with than real world
  - ➢ Easy to check many approaches, parameter values, ...
  - ➢ Flexible to time-scales
  - ➢ Can access un-measurable quantities
- Support safety
  - ➢ Experiments may be dangerous
  - ➢ Operators need to be trained for extreme situations
- Help to gain insight and better understanding

Mathematical Models

Model descriptions

- Transfer functions
- State-space models
- Block diagrams

Notation for continuous-time and discrete-time models

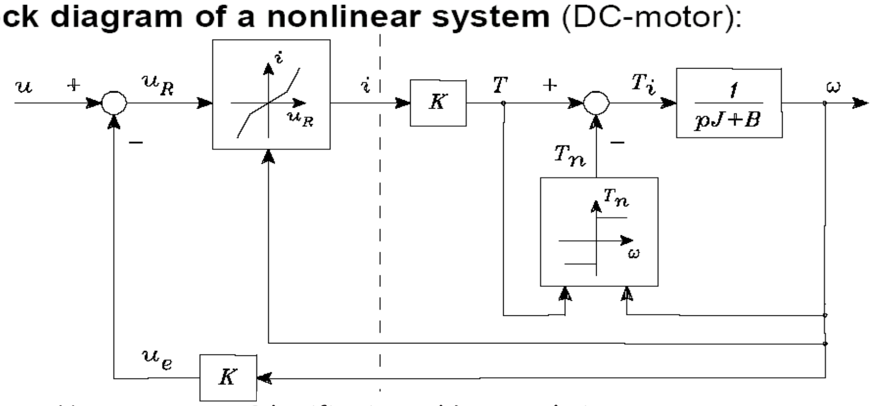
Complex Laplace variable s and differential operator p:

$$\dot{x}(t) = \partial x(t) / \partial t = px(t)$$

Complex z-transform variable z and shift operator q:

$$x(k+1) = qx(k)$$

Block diagram of a nonlinear system (DC-motor):



Type of Models and System Modeling

Models

mathematical – other

parametric – nonparametric

continuous-time – discrete-time

input/output – state-space

linear – nonlinear

dynamic – static

time-invariant – time-varying

SISO – MIMO

Modeling/System Identification

theoretical (physical) – **experimental**

white-box – **grey-box** – black-box

structure determination – parameter estimation

time-domain – frequency-domain

direct – indirect

Types of Models

- Parametric and Non-parametric Models

Many approaches to system identification, depending on model class

– linear/nonlinear

– parametric/nonparametric

Non-parametric methods try to estimate a generic model of a signal or system.

– step responses, impulse responses, frequency responses, etc.

Parametric methods estimate parameters in a user-specified model

– parameters in transfer functions, state-space matrices of given order, etc.

Types of Models

- Linear and Nonlinear Models

The system identification methods are characterized by model type:

A. Linear discrete-time model: Classical system identification

B. Neural network: Strongly non-linear systems with complicated structures – no relation to the actual physical structures/parameters (will not be covered)

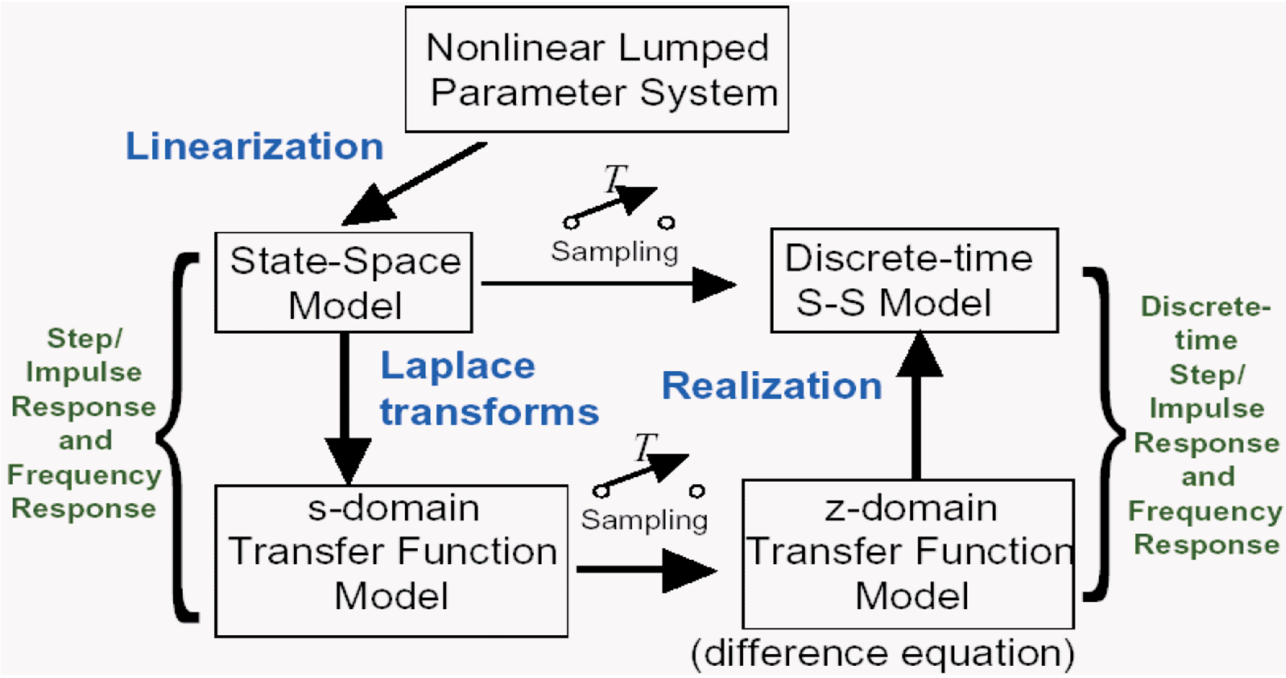
C. General simulation model: Any mathematical model, that can be simulated e.g. with Matlab\Simulink. It requires a realistic physical model structure, typically developed by theoretical modelling

Types of Models – Cont'd

Models can also be classified according to purpose:

- **Models to assist plant design and operation**
  - Detailed, physically based, often non-dynamic models to assist in fixing plant dimensions and other basic parameters
  - Economic models allowing the size and product mix of a projected plant to be selected
  - Economic models to assist decisions on plant renovation
- **Models to assist control system design and operation**
  - Fairly complete dynamic model, valid over a wide range of process operation to assist detailed quantitative design of a control system
  - Simple models based on crude approximation to the plant, but including some economically quantifiable variables, to allow the scope and type of a proposed control system to be decided
  - Reduced dynamic models for use on-line as part of a control system

Systems/Models Representations



How to Build Mathematical Models?

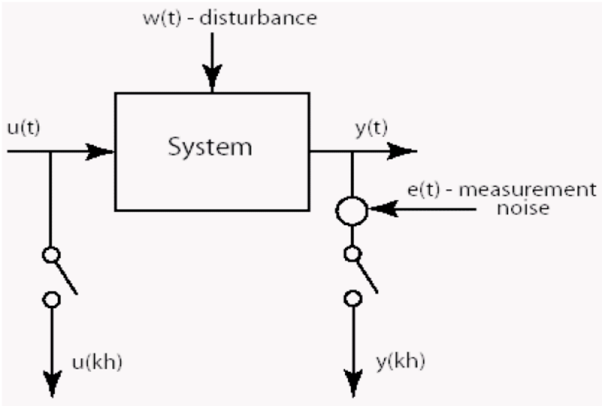
Two basic approaches:

- **Physical modeling**
  - ❑ Use first principles, laws of nature, etc. to model components
  - ❑ Need to understand system and master relevant facts!

- **System identification** - Experimental modeling
  - ❑ Use experiments and observations to deduce model
  - ❑ Need prototype or real system!

Principle of System Identification

Basic Idea: estimate system from measurement of *u*(*t*) and *y*(*t*)



Issues:

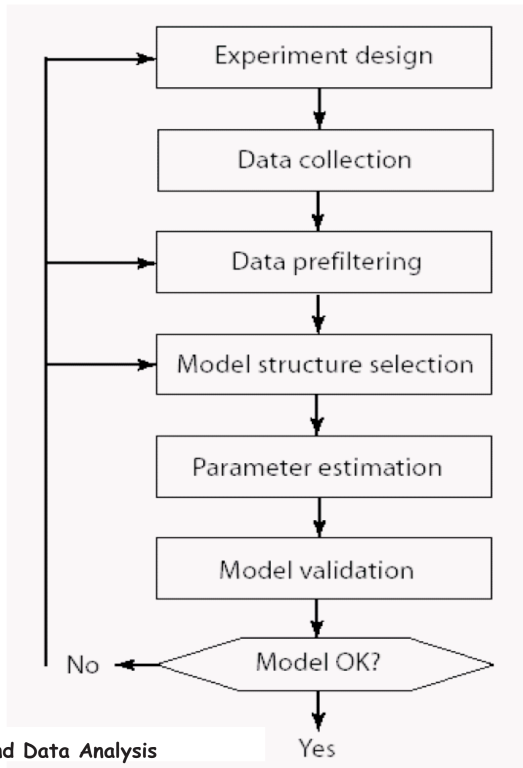
– Choice of sampling frequency, input signal (experimental conditions)

– What class of models – how to model disturbances?

– Estimating model parameters from sampled, finite and noisy data

Procedure of System Identification

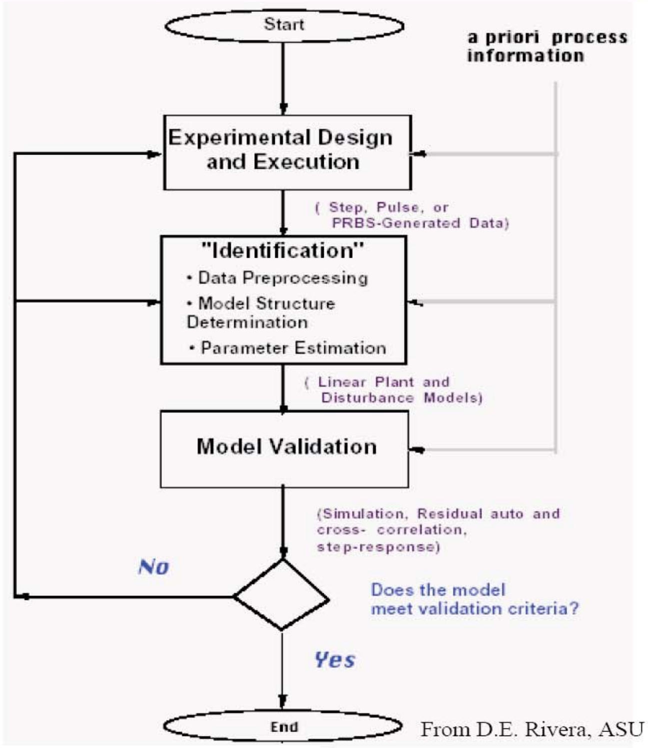
- Experiment design and data collection
- Data preprocessing
- Model structure selection
- Parameter estimation
- Model validation



An iterative procedure !

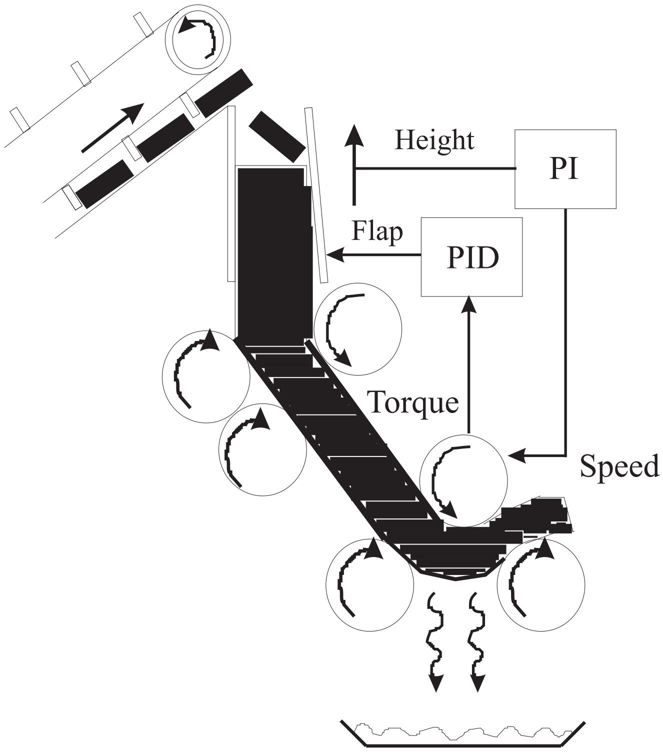
Procedure of System Identification – I

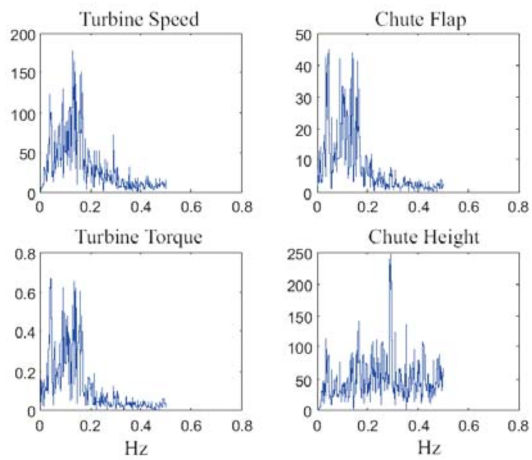
- Experimental design and execution
- Data preprocessing
- Model structure determination
- Parameter estimation
- Model validation

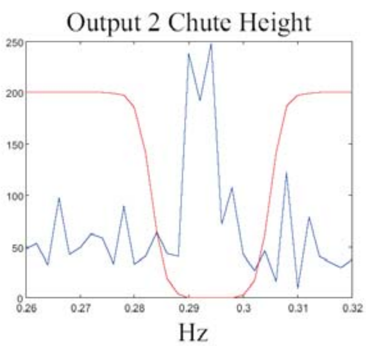


From D.E. Rivera, ASU

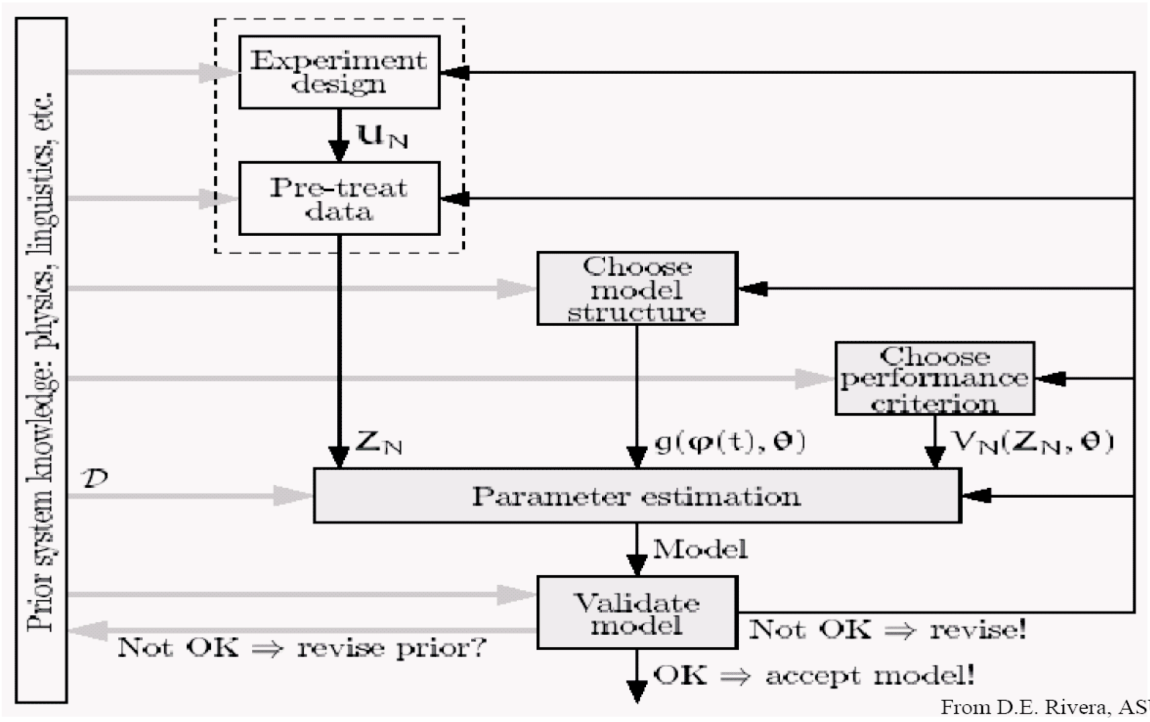
Sugar Cane Crushing Process







Procedure of System Identification – II



From D.E. Rivera, ASU;
Originally from P. Lindskog

Experiments and Data Collection

Often good to use a two-stage approach

1. Preliminary experiments

– step/impulse response tests to get basic understanding of system dynamics

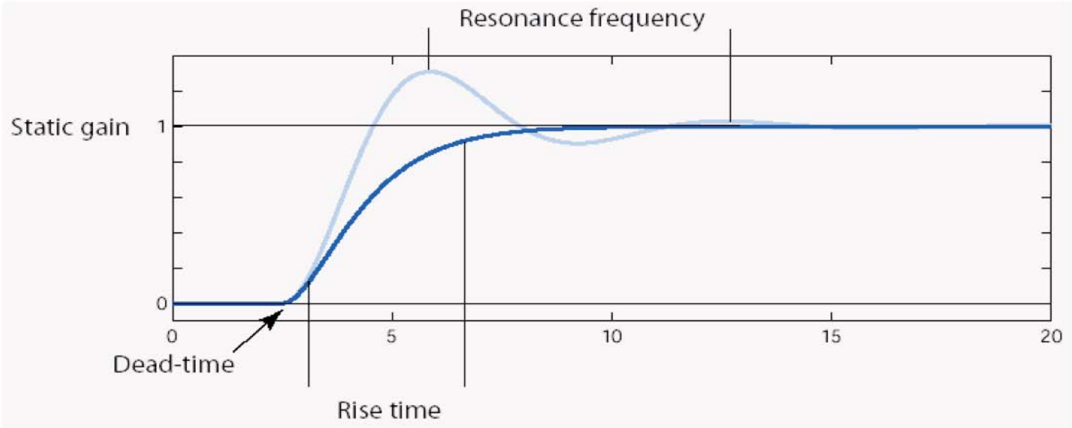
– linearity, static gains, time delays, time constants, sampling interval

2. Data collection for model estimation

– carefully designed experiment to enable good model fit

– operating point, input signal type, number of data points to collect, etc.

Preliminary Experiments: Step Response Experiment



Useful for obtaining qualitative information about system

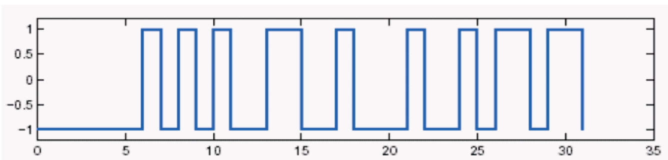
- Indicates dead-times, static gain, time constants and resonance frequency etc.
- Aids sampling time selection (rule-of-thumb: 4-10 sampling points over the rise time)

Designing Experiment for Model Estimation

Input signal should excite all relevant frequencies

– estimated model are more accurate in frequency ranges where input has high energy

– a good choice is often a binary sequence with random "hold times" (*e.g.*, PRBS – Pseudo-Random Binary Sequence)



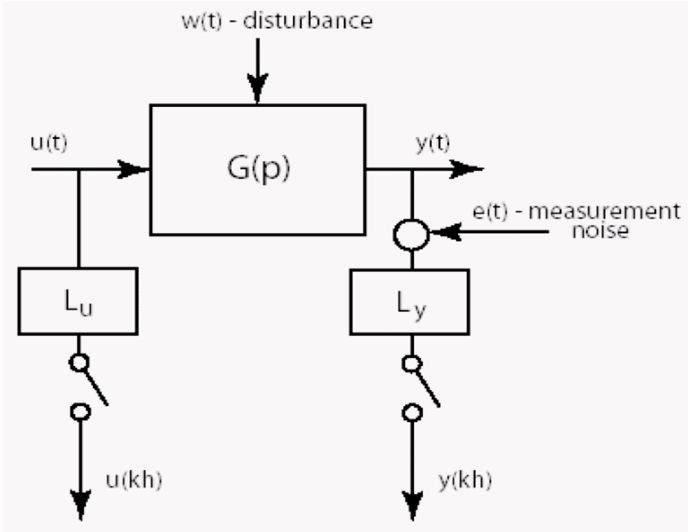
Trade-off in selection of signal amplitude

– large amplitude gives high signal-to-noise ratio (SNR), low parameter variance

– most systems are nonlinear for large input amplitudes

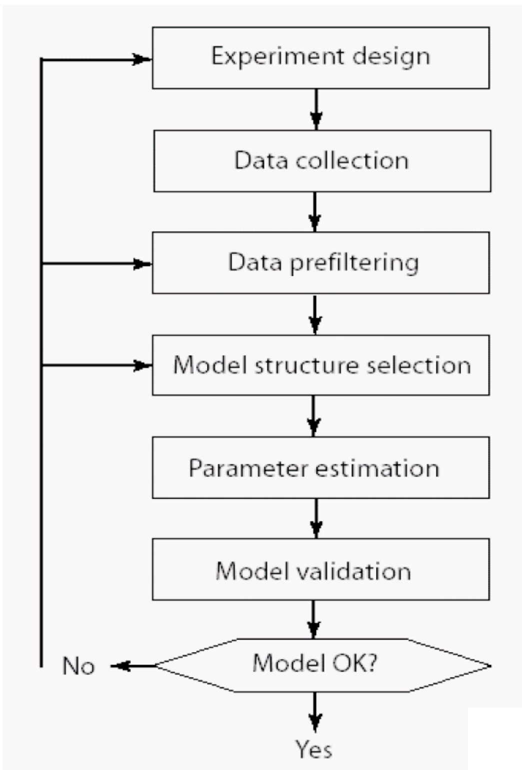
Many pitfalls if estimating a model of a system under closed-loop control !

Data Collection



Sampling time selection and anti-alias filtering are central !

Procedure of System Identification



An iterative procedure !

Prefiltering of Data

Remove

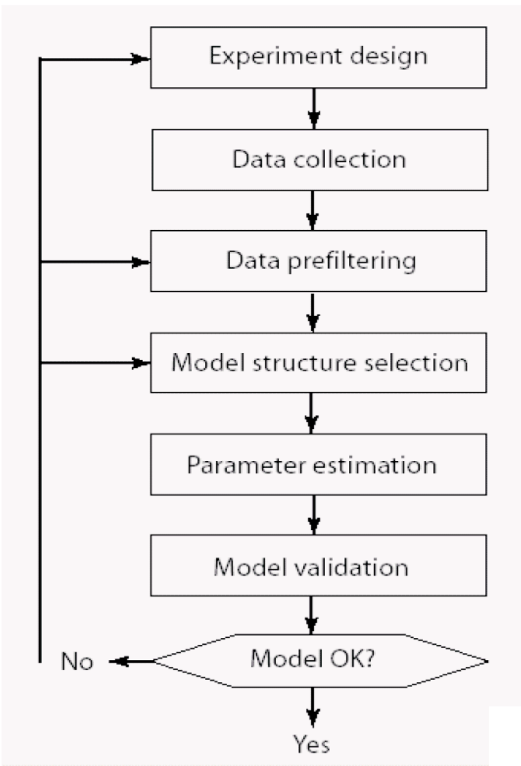
– transients needed to reach desired operating point
– mean values of input and output signals, *i.e.*, work with

$$\Delta u[t] = u[t] - \frac{1}{N}\sum_{t=1}^{N} u[t]$$

$$\Delta y[t] = y[t] - \frac{1}{N}\sum_{t=1}^{N} y[t]$$

– trends (use `detrend` in MATLAB)
– outliers (“obviously erroneous data points”)

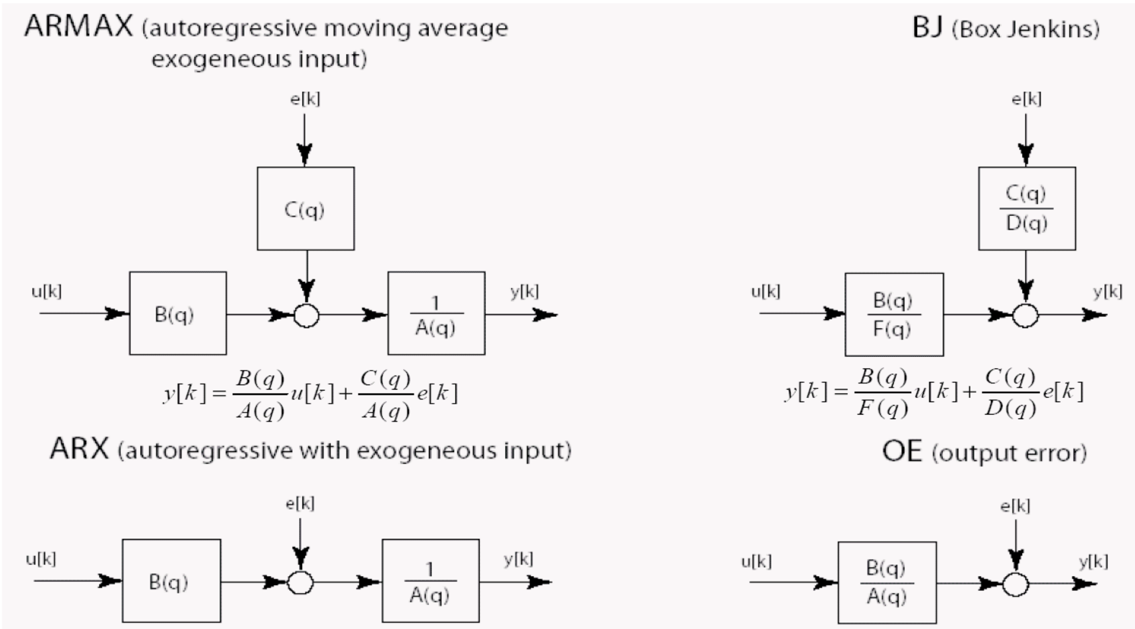
Procedure of System Identification



An iterative procedure !

Model Structures

Model structures commonly used (BJ includes all others as special cases)



ARMAX: $$y[k]=\frac{B(q)}{A(q)}u[k]+\frac{C(q)}{A(q)}e[k]$$

BJ: $$y[k]=\frac{B(q)}{F(q)}u[k]+\frac{C(q)}{D(q)}e[k]$$

ARX: $$y[k]=\frac{B(q)}{A(q)}u[k]+\frac{1}{A(q)}e[k]$$

OE: $$y[k]=\frac{B(q)}{A(q)}u[k]+e[k]$$

Model Structures - Cont´d

- **Model structures Based on Input-Output**

Model	$\tilde{p}(q)$	$\tilde{p}_e(q)$
ARX	$\frac{B(q)}{A(q)}$	$\frac{1}{A(q)}$
ARMAX	$\frac{B(q)}{A(q)}$	$\frac{C(q)}{A(q)}$
FIR	$B(q)$	1
Box-Jenkins	$\frac{B(q)}{F(q)}$	$\frac{C(q)}{D(q)}$
Output Error	$\frac{B(q)}{F(q)}$	1

$$A(q)y[k]=\frac{B(q)}{F(q)}u[k]+\frac{C(q)}{D(q)}e[k] \quad \text{or} \quad y[k]=\tilde{p}(q)u[k]+\tilde{p}_e(q)e[k]$$

- **Model structures Based on State-Space Representation**

$$\begin{aligned} x[k+1] &= Ax[k]+Bu[k] \\ y[k+1] &= Cx[k+1]+Du[k+1] \end{aligned} \quad \text{or} \quad \begin{aligned} x[k+1] &= A(\theta)x[k]+B(\theta)u[k] \\ y[k+1] &= C(\theta)x[k+1]+D(\theta)u[k+1] \end{aligned}$$

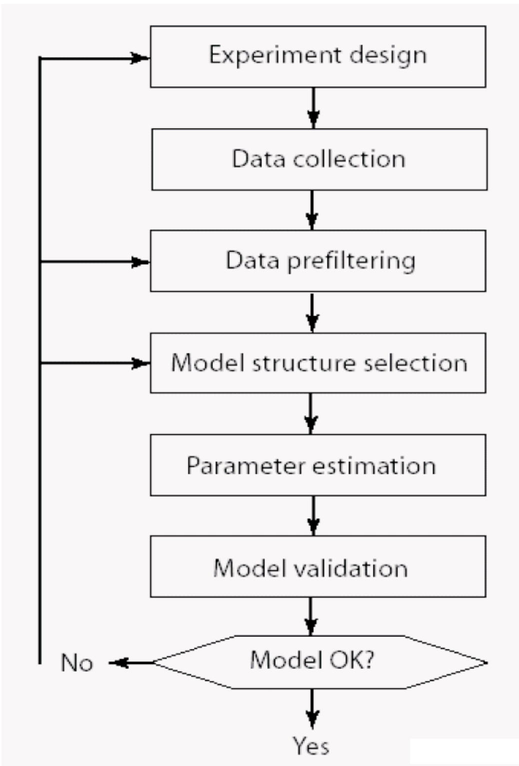
Choice of Model Structure

1. Start with non-parametric estimates (correlation analysis, spectral estimation)
   – give information about model order and important frequency regions
2. Prefilter input/output data to emphasize important frequency ranges
3. Begin with ARX (AutoRegressive with eXogeneous input) models
4. Select model orders via
   – cross-validation (simulate model and compare with new data)
   – Akaike's Information Criterion (AIC), *i*.e., pick the model that minimizes

$$(1+2\frac{d}{N})\sum_{t=1}^{N}\varepsilon[t;\theta]^2$$

(where d is the number of estimated parameters in the model)

Procedure of System Identification



An iterative procedure !

Nonparametric Estimation Methods

Nonparametric methods

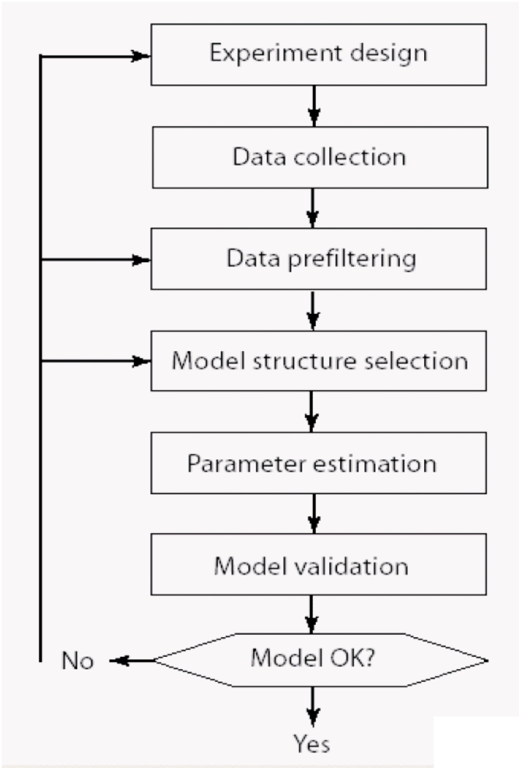
- Transient response
- Correlation analysis
- Frequency responses analysis and Fourier analysis
- Spectral analysis

❑ Discussed in the “Automatica I (Laboratorio)” course, will not elaborated further in this course

Parametric Estimation Methods

- Non-recursive/Batch (off-line) methods
  - Linear regression and (block) least squares methods
  - Prediction error methods
  - Instrumental variable methods
  - Subspace methods *(If possible, few details)*
- Recursive (on-line) methods
  - Recursive Least Squares (RLS) methods
  - Forgetting factor techniques and time-varying svstems identification methods

Procedure of System Identification



An iterative procedure !

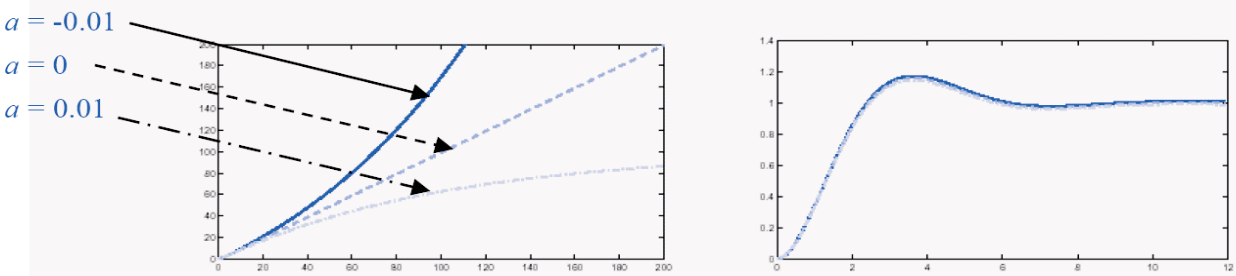
Model Validation

A critical evaluation: “is model good enough”?
– typically depends on the purpose of the model

Example

$$G(s) = \frac{1}{(s+1)(s+a)}$$

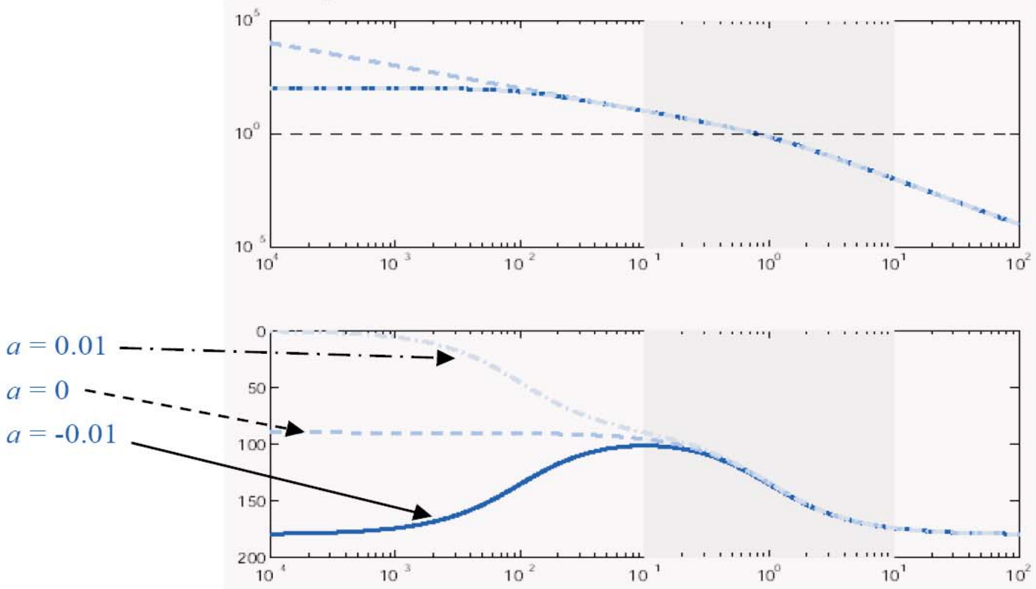
Open- and closed-loop responses for $a = -0.01,\ 0,\ 0.01$



Insufficient for open-loop prediction, good enough for closed-loop control.

Model Validation – cont'd

- Bode diagrams reveal why model is good enough for closed-loop control



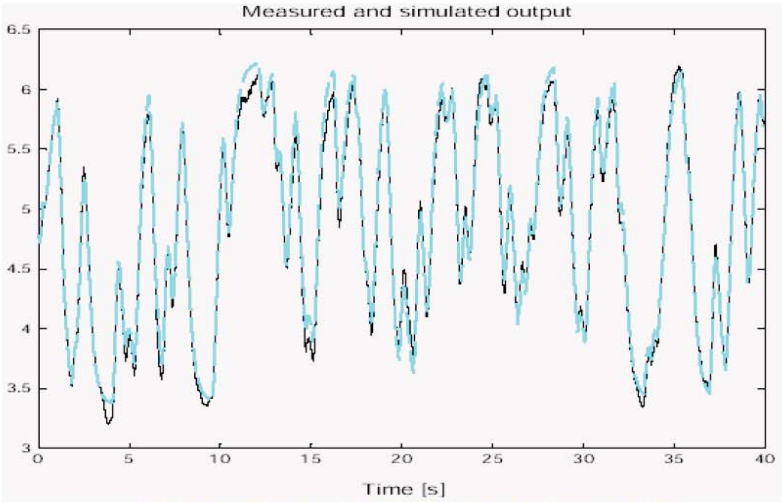
- Different low-frequency behavior, similar responses around cross-over frequency

Principle of Model Validation

1. Compare model simulation/prediction with real data – in time domain
2. Compare estimated model's frequency response and spectral analysis result – in frequency domain
3. Perform statistical tests on prediction errors

Validation: simulation and prediction

- Split data into two parts: one for estimation and one for validation
- Apply input signal in validation data set to estimated model
- Compare simulated output with output stored in validation data set



Statistical Model Validation

If we fit the parameters of the model

$$y[t] = G(q;\theta)u[t] + H(q;\theta)e[t]$$

to data, the *residuals*

$$\varepsilon[t] = H(q;\theta)^{-1}\{y[t] - G(q;\theta)u[t]\}$$

represent a disturbance that explains mismatch between model and observed data.

If the model is correct, the residuals should be
- – white, and
- – uncorrelated with u

Statistical Model Validation – cont'd

To test if the residuals $\varepsilon[t]$ are **white**, we compute the auto-covariance function

$$\hat{R}_{\varepsilon}(\tau) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon[t]\varepsilon[t+\tau]$$

and verify that its components lie within a 95% confidence region around zero.

– large components indicate un-modelled dynamics

Independence tested by verifying that cross-correlation function

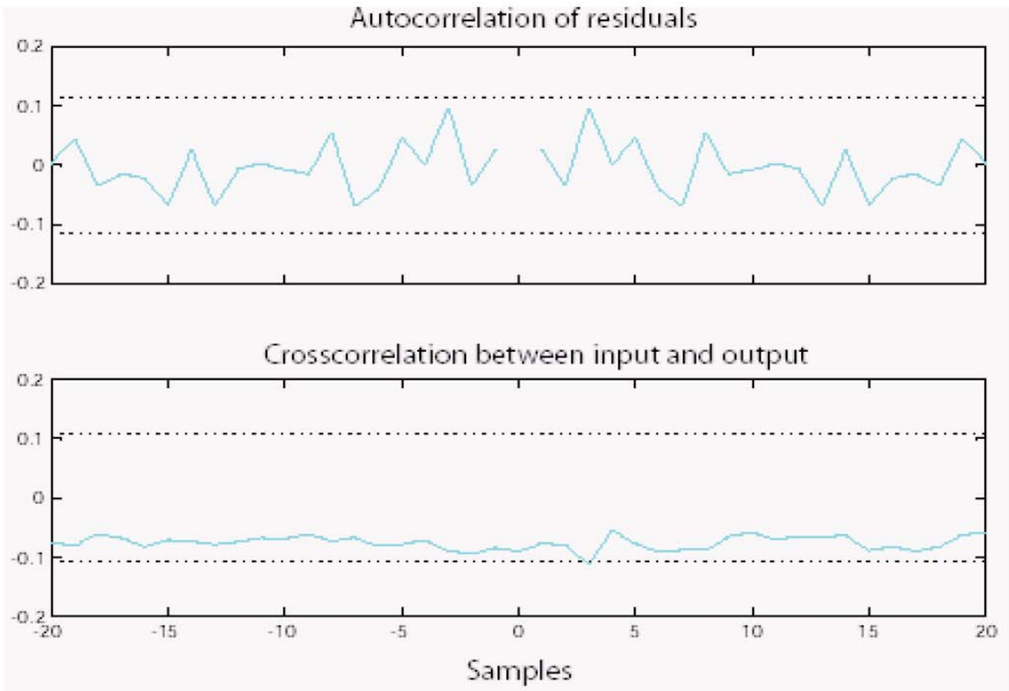
$$\hat{R}_{\varepsilon u}(\tau) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon[t+\tau]u[t]$$

lie within a 95% confidence region around zero.

– large components indicate un-modelled dynamics,

– $\hat{R}_{\varepsilon u}(\tau)$ nonzero for $\tau < 0$ (non-causality) indicate the presence of feedback

Statistical Model Validation – cont'd



Software Tools
- MATLAB Toolbox: System Identification

```
>> help ident
  System Identification Toolbox.
  Version 5.0.1 (R12.1)  18-May-2001

 Simulation and prediction.
   predict    - M-step ahead prediction.
   pe         - Compute prediction errors.
   sim        - Simulate a given system.

 Data manipulation.
   iddata     - Construct a data object.
   detrend    - Remove trends from data sets.
   idfilt     - Filter data through Butterworth filters.
   idinput    - Generates input signals for identification.
   merge      - Merge several experiments.
   misdata    - Estimate and replace missing input and output data.
   resample   - Resamples data by decimation and interpolation.
```


Software Tools
- MATLAB Toolbox: System Identification – cont'd

```
 Nonparametric estimation.
   covf       - Covariance function estimate for a data matrix.
   cra        - Correlation analysis.
   etfe       - Empirical Transfer Function Estimate and Periodogram.
   impulse    - Direct estimation of impulse response.
   spa        - Spectral analysis.
   step       - Direct estimation of step response.

 Parameter estimation.
   ar         - AR-models of signals using various approaches.
   armax      - Prediction error estimate of an ARMAX model.
   arx        - LS-estimate of ARX-models.
   bj         - Prediction error estimate of a Box-Jenkins model.
   ivar       - IV-estimates for the AR-part of a scalar time series.
   iv4        - Approximately optimal IV-estimates for ARX-models.
   n4sid      - State-space model estimation using a sub-space method.
   oe         - Prediction error estimate of an output-error model.
   pem        - Prediction error estimate of a general linear model.
```

Software Tools
- MATLAB Toolbox: System Identification – cont'd

Model structure creation.

idpoly - Construct a model object from given polynomials.
idss - Construct a state space model object.
idarx - Construct a multivariable ARX model object.
idgrey - Construct a user-parameterized model object.

Model conversions.

arxdata - Convert a model to its ARX-matrices (if applicable).
polydata - Polynomials associated with a given model.
ssdata - IDMODEL conversion to state-space.
tfdata - IDMODEL conversion to transfer function.
zpkdata - Zeros, poles, static gains and their standard deviations.
idfrd - Model's frequency function, along with its covariance.
idmodred - Reduce a model to lower order.
c2d, d2c - Continuous/discrete transformations.
ss, tf, zpk, frd - Transformations to the LTI-objects of the CSTB.
Most CSTB conversion routines also apply to the model objects of the Identification Toolbox.

Software Tools
- MATLAB Toolbox: System Identification – cont'd

Model presentation.

bode - Bode diagram of a transfer function or spectrum (with uncertainty regions).
ffplot - Frequency functions (with uncertainty regions).
plot - Input - output data for data objects.
present - Display the model with uncertainties.
pzmap - Zeros and poles (with uncertainty regions).
nyquist - Nyquist diagram of a transfer function (with uncertainty regions).
view - The LTI viewer (with the Control Systems Toolbox for model objects).

Model validation.

compare - Compare the simulated/predicted output with the measured output.
pe - Prediction errors.
predict - M-step ahead prediction.
resid - Compute and test the residuals associated with a model.
sim - Simulate a given system (with uncertainty).

Model structure selection.

aic, fpe - Compute Akaike's information and final prediction criteria
arxstruc - Loss functions for families of ARX-models.
selstruc - Select model structures according to various criteria.
struc - Typical structure matrices for ARXSTRUC.

Software Tools
- MATLAB Toolbox: System Identification – cont'd

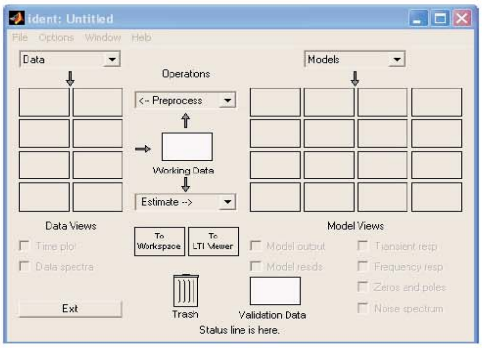
Practice yourself using Matlab System Identification toolbox demonstrations: "iddemo"

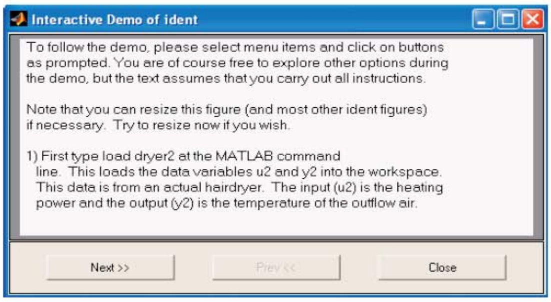
>> iddemo

The SYSTEM IDENTIFICATION TOOLBOX is an analysis module that contains tools for building mathematical models of dynamical systems, based upon observed input-output data.
The toolbox contains both PARAMETRIC and NON-PARAMETRIC MODELING methods.

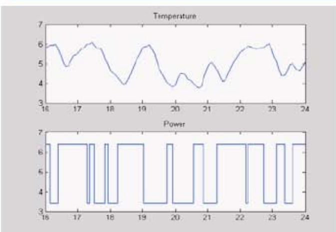
Identification Toolbox demonstrations:

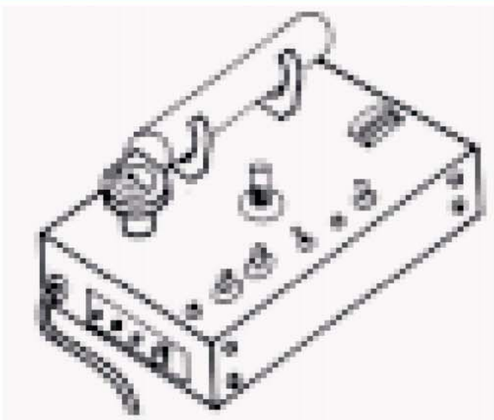
1) The Graphical User Interface (ident): A guided Tour.
2) Build simple models from real laboratory process data.
3) Compare different identification methods.
4) Data and model objects in the Toolbox.
5) Dealing with multivariable systems.
6) Building structured and user-defined models.
7) Model structure determination case study.
8) How to deal with multiple experiments.
9) Spectrum estimation (Marple's test case).
10) Adaptive/Recursive algorithms.
11) Use of SIMULINK and continuous time models.
12) Case studies.



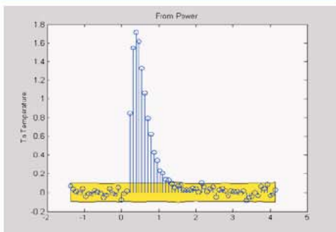


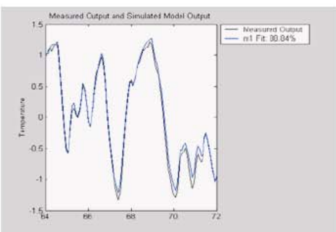
A System Identification Example: Hairdryer

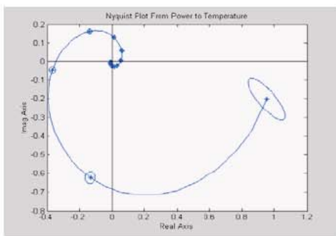


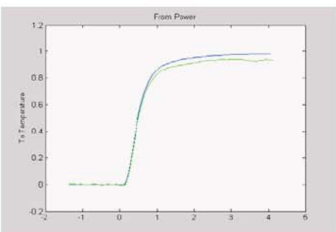


Feedback's Process Trainer PT326



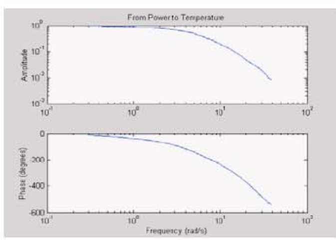




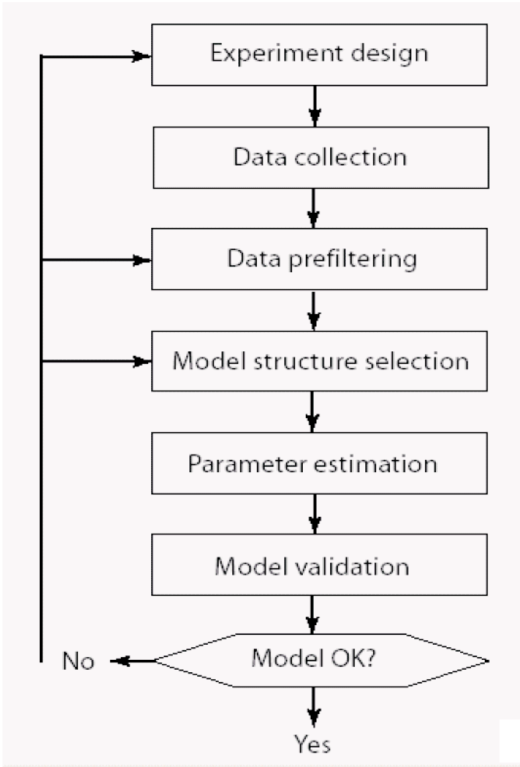


"Hairdryer" process: input is the voltage over the heating device; output is outlet temperature

Matlab: "iddemo" (demonstration 2)



Main Focus in This Course



An iterative procedure !

Reading and Exercise

- **Reading**: Textbook, Chapter 1; Sections 4.1-4.3
- **Further Reading** *(Master's Theses)*:
  - ❑ L. Ljung, *From Data to Model: A Guided Tour of System Identification*, Report No. LiTH-ISY-R-1652, Linköping University, Sweden, 1994.
- **Exercise**: None

Exams Procedure

- Data Selection and System Identification
- System Identification Toolbox in Matlab
  - Report preparation
- Oral examination

Lecture 1:

System Identification and Data Analysis

Any question, comment or suggestion?