General Overview

- Growing demand for higher reliability
- Actuators & sensors are the most important components
  - hardware & software (analytical) redundancy
- Hardware redundancy methods use multiple lanes of sensors, computers & software to measure and/or control a particular variable
  - multiple hardware redundancy is harder to achieve
FDI Overview

- Analytical redundancy makes use of a mathematical model of the monitored process
  - *model-based* approach to Fault Detection & Isolation (FDI)
  - The model-based FDI is normally implemented in software form as a computer algorithm

- Model-based methods use a model of the monitored system to produce residuals
  - The system cannot be described accurately by a mathematical model

Model-Based FDI

- In real complex systems modelling uncertainty arises inevitably for example process noise, parameter variations & modelling errors
  - The detection of incipient faults presents a challenge to model-based FDI techniques
  - Unseparable mixture between fault effects & modelling uncertainty
Robust Model–Based FDI

- Optimisation to minimise the effect of modelling uncertainty, whilst maximising some fault effects
- Intelligent techniques, adaptive methods
- Robust FDI is still an open problem for further research

Presentation Outline

- Fault Detection & Isolation
  - Linear multivariable systems with additive faults & disturbances
- Input–output polynomial forms
- Residual generator subspace basis ↔ dynamic filter computation
  - Analytical Derivation, Fault & noise sensitivity analysis
- Application: sensor & actuator faults of dynamic processes
  - Linear & linearised model
Introduction: System Modelling

- **Fault diagnosis for dynamic processes**
  - ⇒ disturbance de-coupling techniques

- **Input–output descriptions of the monitored system**
  - ⇒ model disturbance term takes into account system unknown inputs

Introduction: Fault Diagnosis

- **Set of parity relations insensitive to the disturbance term**
  - ⇒ residual or symptom signals

- **Simulated process: power plant & small aircraft**
  - ⇒ sensor & actuator faults
Analytical Redundancy & Model–based Approach

- **Main problem:** modelling uncertainties
  - unavoidable in real industrial systems
- **System description** → input–output linear model
  - by modelling or identification procedures
  - the disturbance term describes unknown (or non–measurable) inputs of the real process

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Model–based Approach

- **Parity relations design for residual generation**
- **Detection of faults affecting input & output process measurements**
- **Residual generator:** insensitive to disturbance signals
Presentation Topics (1)

- The system under diagnosis is modelled in terms of input–output polynomial description
- The design of disturbance de–coupled residual generators is reduced to the determination of the null–space of a specific polynomial matrix
  ⇒ The use of input–output forms allows to design the analytical description for the disturbance de–coupled residual generators

Presentation Topics (2)

- These dynamic fault detection filters, organised into bank structures, are able to achieve Fault Isolation properties
  ⇒ An appropriate choice of their parameters allows to maximise robustness with respect to both measurement noise & modelling errors, while optimising fault sensitivity characteristics
- The proposed FDI approach has been applied to 2 dynamic process models:
  ⇒ (i) Power Plant of “Pont sur Sambre”
  ⇒ (ii) Piper PA30
Presentation Topics (3)

- The residual generators have been designed on the basis of the linear & linearised models
  ⇒ Experiments with data from linear & non–linear simulators implemented in Matlab/Simulink® environment

- An important aspect of the approach to FDI suggested is the simplicity of structure of the technique used to generate the residual functions for FDI
  ⇒ In comparison with traditional schemes e.g. based on banks of unknown input observers (UIO) & Kalman filters

Talk Structure (1)

- Mathematical description of the monitored system is outlined
- The approach exploited for the design of residual generators is described
- Structural characteristics of such filters are also explained
  ⇒ How to achieve disturbance de–coupling, sensitivity optimisation of the residual functions & robustness with respect to measurement noise & modelling errors
Talk Structure (2)

- Problem of the design of banks of residual generators for the isolation of faults affecting the input & the output sensors
- Application to 2 dynamic process models with some numerical results

General Scenario

- Monitored system

  ![Diagram]

  - Faults
  - Actuator
  - Process
  - Sensor
  - Input
  - Output
  - Disturbance

  ⇒ additive fault & disturbance occurrence

  ⇒ fault-free & faulty data sequences
Mathematical Description

\[ P(s) y(t) = Q(s) u(t) \]

- \( s \) is the derivative operator

- \( P(s) \) & \( Q(s) \) are polynomial matrices with dimension \( (m \times m) \) & \( (m \times \ell) \) respectively, with \( P(s) \) nonsingular.

- \( u(t) \in \mathbb{R}^\ell \) & \( y(t) \in \mathbb{R}^m \), the input & output vectors of the considered multivariable system

- \( u(t) \equiv \mathcal{L}[u(t)](s) \) & \( y(t) \equiv \mathcal{L}[y(t)](s) \)

Model Properties

- **Models of type** \( P(s) y(t) = Q(s) u(t) \) **can be frequently found in practice**
  
  ⇒ applying well–known physical laws

  ⇒ for describing the input–output dynamical links

- **powerful tool when the knowledge of the system state does not play a direct role**
Model Properties

\[ P(s) y(t) = Q(s) u(t) \]

- Powerful tool when the knowledge of the system state does not play a direct role
  \[ \Rightarrow \text{see, e.g. residual generator design, identification, de–coupling, output controllability,...} \]

- Algorithms to transform state–space models to equivalent input–output polynomial representations & vice versa are available [Beghelli, Guidorzi, Castaldi, Soverini]

Mathematical Description

\[ P(s) y(t) = \begin{bmatrix} Q_c(s) & Q_d(s) & Q_f(s) \end{bmatrix} \begin{bmatrix} c(t) \\ d(t) \\ f(t) \end{bmatrix}, \]

- The equivalent representation \( \{ \hat{P}(s), \hat{Q}(s) \} \) is a canonical input–output form
- Link between \( \{ P(s), Q(s) \} \) & \( \{ \hat{P}(s), \hat{Q}(s) \} \) via an unimodular matrix \( M(s) \)
- \( c(t) \equiv \mathcal{L}[c(t)](s), \ d(t) \equiv \mathcal{L}[d(t)](s) \) & \( f(t) \equiv \mathcal{L}[f(t)](s) \)
Model Description for the Monitored System

\[ d(t) \]
\[ Q_d(s) \]
\[ c(t) \]
\[ Q_c(s) \]
\[ + \]
\[ P^{-1}(s) \]
\[ y(t) \]
\[ Q_f(s) \]
\[ f(t) \]

Input–Output Canonical Form Properties

\[
\begin{align*}
\deg \tilde{p}_{ii}(s) &> \deg \tilde{p}_{ji}(s) \quad i \neq j \\
\deg \tilde{p}_{ii}(s) &> \deg \tilde{p}_{ij}(s) \quad j > i \\
\deg \tilde{p}_{ii}(s) &\geq \deg \tilde{q}_{ij}(s) \\
\deg \tilde{p}_{ii}(s) &\geq \deg \tilde{q}_{ij}(s)
\end{align*}
\]

- with the polynomials \( \tilde{p}_{ii}(s) \) monic.
- Integers \( \nu_i = \deg \tilde{p}_{ii} \) \((i = 1, \ldots, m)\) equal the corresponding row-degrees.
- Integers \( \nu_i \) are the ordered set of Kronecker invariants associated to the pair \( \{ \tilde{A}, \tilde{C} \} \) of every observable realization of \( \{ \tilde{P}(s), \tilde{Q}(s) \} \).
State–Space Canonical Form Properties

Canonical state–space \((\tilde{A}, \tilde{C})\) models:

\[
\tilde{A} = [\tilde{A}_{ij}] , \text{ with } \tilde{A}_{ii} = \begin{bmatrix}
0 & 1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1 \\
\end{bmatrix}_{(\nu_1 \times \nu_1)} \\
\times_{ii1} \times_{ii2} \ldots \times_{iiv_i}
\]

\[
\tilde{A}_{ij} = \begin{bmatrix}
0 & \ldots & \ldots & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \\
0 & \ldots & \ldots & \ldots & 0 \\
\end{bmatrix}_{(\nu_1 \times \nu_2)} \\
\times_{ij1} \ldots \times_{ij
v_ij} 0 & \ldots & 0
\]

\[
\tilde{C} = \begin{bmatrix}
1 & 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\
0 & \ldots & 0 & 1 & 0 & \ldots & \ldots & \ldots & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \ldots & \ldots & \ldots & \ldots & 0 & 1 & 0 & \ldots & 0
\end{bmatrix}
\]

⇒ where the 1’s entries in the matrix \(\tilde{C}\) are in the column 1, \((\nu_1 + 1), \ldots, (\nu_1 + \ldots + \nu_{m-1} + 1)\)
Filter Design for Fault Diagnosis

- A general linear residual generator is a filter of type:

\[ R(s) r(t) = S_y(s) y(t) + S_c(s) c(t) \]

⇒ \( r(t) \) signal is a scalar

⇒ Faults are neglected here!

Residual Generator Computation Problem

⇒ Design the residual generator (\( r(t) \) scalar)

\[ R(s) r(t) = S_y(s) y(t) + S_c(s) c(t) \]

⇒ for the input–output model (fault–free conditions)

\[ P(s) y(t) - Q_c(s) c(t) = Q_d(s) d(t) \]
Residual Generator Computation Problem

\[ \Rightarrow L(s) \text{ is a polynomial row belonging to the left null–space of } Q_d(s) \]

\[ \Rightarrow \text{Left null–space of } Q_d(s) \text{ is } \mathcal{N}_L(Q_d(s)) \]

\[ L(s) \in \mathcal{N}_L(Q_d(s)) \]

\[ \Rightarrow i.e. \quad L(s) Q_d(s) = 0 \]

\[ \Rightarrow e.g. \quad 2 \text{ outputs } & 1 \text{ disturbance} \]

Residual Generator Formulation (with faults)

\[ P(s) y(t) - Q_c(s) c(t) - Q_f(s) f(t) = Q_d(s) d(t) \]

\[ L(s) P(s) y(t) - L(s) Q_c(s) c(t) = L(s) Q_f(s) f(t) \]

\[
\left\{
\begin{array}{l}
S_y(s) = L(s) P(s) \\
S_c(s) = -L(s) Q_c(s) \\
R(s) = (1 + \tau_1 s)(1 + \tau_2 s) \ldots (1 + \tau_{nf} s) = a_1 s^{nf} + a_2 s^{nf-1} + \cdots + a_{nf} s + 1
\end{array}
\right.
\]
Residual Generator Formulation

- $n_f$ is the maximal row–degree of the pair $\{L(s) P(s), L(s) Q_c(s)\}$.

- Without faults: $R(s) r(t) = L(s) P(s) y(t) - L(s) Q_c(s) c(t) = 0$

- With faults: $R(s) r(t) = L(s) P(s) y(t) - L(s) Q_c(s) c(t) = L(s) Q_f(s) f(t)$

Residual Generator Properties

**Bounds for the order $n_f$ of the residual generator**

- $n_f$ is the maximal row–degree of the pair $\{L(s) P(s), L(s) Q_c(s)\}$
- $n_f$ is the degree of $R(s)$ (filter causality)

\[ \nu_{\min} \leq n_f \leq (\ell_d + 1) \nu_{\max} \]

- $\nu_{\min}$ & $\nu_{\max}$ are the minimal & the greatest Kronecker invariant, respectively.
- [Frisk & Niberg, Bonfè, et al.]
Fault Description

Fault modelling

\[
\begin{align*}
\dot{c}(t) & = c(t) + f_c(t) \\
\dot{y}(t) & = y(t) + f_y(t)
\end{align*}
\]

\[\Rightarrow f_c(t) \text{ & } f_y(t): \text{ actuator & sensor additive faults}\]

\[\Rightarrow \text{ e.g. step, ramp, intermittent signals}\]

\[c(t), y(t): \text{ fault–free signals}\]

\[c^*(t), y^*(t): \text{ input & output measurements}\]
Fault Modelling

\[
\begin{align*}
  c^*(t) &= c(t) + f_c(t) \\
  y^*(t) &= y(t) + f_y(t)
\end{align*}
\]

\[R(s) r(t) = L(s) Q_f(s) f(t) \leftrightarrow \text{ideal conditions!}\]

with \(f(t) = \begin{bmatrix} f_c(t) \\ f_o(t) \end{bmatrix}\)

\[Q_f(s) = [Q_c(s) - P(s)]\]

---

Residual function \(r(t)\) for Fault Detection

- **Fault-free & faulty situations**
- **Residual function \(r(t)\) comparison**
  
  ⇒ fixed threshold \(\varepsilon\)
  
  ⇒ threshold logic:

\[
\begin{cases}
  |r(t)| & \leq \varepsilon \quad \text{for fault-free case,} \\
  |r(t)| & > \varepsilon \quad \text{for faulty cases.}
\end{cases}
\]

- **Analysis of different residual functions \(r_i(t)\)**
Residual Generator Design

\[ R(s) r(t) = \underbrace{L(s) P(s) y(t) - L(s) Q_c(s) c(t)}_{\text{modelling error} \neq 0} + \underbrace{L(s) Q_f(s) f(t)}_{\text{faulty case}} = 0 \]

\[ R(s) r(t) = \underbrace{L(s) Q_f(s) f(t)}_{\text{faulty case}} \]

- **Ideal conditions**
  - \( \Rightarrow \) Easy solution

- **Optimisation Approach (real conditions):**
  \[ R(s) r(t) = \underbrace{L(s) P(s) y(t) - L(s) Q_c(s) c(t)}_{\text{modelling error} \neq 0} + \underbrace{L(s) Q_f(s) f(t)}_{\text{faulty case}} \]

  \( \Rightarrow \) \( R(s) \) & \( L(s) \) \( \leftrightarrow \) optimal selection

  \[ R(s) = (1 + \tau_1 s)(1 + \tau_2 s) \cdots (1 + \tau_{n_f}) \]

- **Fault sensitivity maximisation:** \( \max \) of \( R^{-1}(s) L(s) Q_f(s) f(t) \)
Residual Generator Parameter Optimisation: \( L(s) \)

- **Parameters of \( L(s) \)**

  \[
  b_i(s) (i = 1, 2, \ldots, m - \ell_d) \text{ rows of a basis } \mathcal{B}_{(m-\ell_d) \times m}(s) \text{ of the } \mathcal{N}_\ell(Q_d(s))
  \]

  \[
  \Rightarrow \text{ Assumption: } m - \ell_d > 1
  \]

  \[
  \Rightarrow \text{ then, } L(s) = \sum_{i=1}^{m-\ell_d} k_i b_i(s) \text{ & } k_i \text{ maximising:}
  \]

\[
\lim_{s \to 0} \frac{1}{R(s)} \left[ \sum_{i=1}^{m-\ell_d} k_i b_i(s) \right] Q_f(s) = \left[ \sum_{i=1}^{m-\ell_d} k_i b_i(0) \right] Q_f(0) \text{ with } \sum_{i=1}^{m-\ell_d} k_i^2 = 1
\]

- **Fault \( f(t) \) step–function of magnitude \( F \)**

\[
\lim_{t \to \infty} r(t) = \lim_{s \to 0} s \frac{L(s)Q_f(s) F}{R(s)} = \left[ \sum_{i=1}^{m-\ell_d} k_i b_i(0) \right] Q_f(0) F.
\]
Residual Generator Parameter Optimisation: $R(s)$

- **Location of the roots of the polynomial $R(s)$**
  - Influences the transient characteristics (maximum overshoot, delay time, rise time, settling time, etc.) of the fault detection filter
  - Optimisation of fault detection time, false alarm & missed fault rates

\[
\|G_f(j\omega)\|^2 \frac{1}{\|G_r(j\omega)\|^2} = 1, \text{(where } \omega \text{ belongs to a given frequency range)}
\]

⇒ reference transfer function $G_r(s)$

⇒ $G_f(s) = L(s)Q_f(s)/R(s)$
Fault Isolation Introduction

- **After fault detection...**
- **Problem of the design of banks of residual generators**
  - Isolation of faults affecting input & output sensors
  - The disturbance de–coupling method suggested previously is exploited
  - It is assumed that $m > \ell_d + 1$

Input Sensor Fault Isolation

Scheme for input sensor fault isolation.
The number of these generators is equal to the number $\ell_c$ of system control inputs

- The $i$–th device ($i = 1, \ldots, \ell_c$) is driven by all but the $i$–th input & by all the outputs of the system
- A fault on the $i$–th input sensor affects all but the $i$–th residual generator

\[ c^*i(t) \text{ represents the } \ell_c - 1 \text{ dimensional vector obtained by deleting from } c^*(t) \text{ the } i\text{–th component} \]

\[ c^*(t) = c(t) + f_{ci}(t) \]

\[ \Rightarrow \text{ with } f_{ci}(t) = \begin{bmatrix} 0 & \ldots & 0 & h_{ci}(t) & 0 & \ldots & 0 \end{bmatrix}^T \]

\[ \Rightarrow c^{*i}(t) = c^i(t) \text{ when the fault on the } i\text{–th input sensor } h_{ci}(t) \text{ is considered} \]
In these conditions:

\[ P(s) y(t) = Q_c(s) c(t) + Q_d(s) d(t) + q_{ci}(s) h_{ci}(t), \]

\[ q_{ci}(s) \text{ represents the } i \text{–th column of the matrix } Q_c(s) \]

By multiplying by the matrix \( L_{ci}(s) \)

\[ L_{ci}(s) \text{ is a row vector belonging to the basis for the left null space of the matrix } [Q_d(s) \mid q_{ci}(s)] \]

If \( L_{ci}(s) \) is a row vector belonging to the basis for the left null space of the matrix \([Q_d(s) \mid q_{ci}(s)]\)

\[ Q^i_c(s) \text{ is the matrix obtained by deleting from } Q_c(s) \text{ the } i \text{–th column:} \]

The \( i \)–th filter becomes:

\[ R_{ci}(s) r_{ci}(t) = L_{ci}(s) P(s) y(t) - L_{ci}(s) Q^i_c(s) c^{*i}(t) = 0, \]

\[ \Rightarrow \text{ while, for the } j \text{–th filter, with } j \neq i: \]

\[ R_{cj}(s) r_{cj}(t) = L_{cj}(s) P(s) y(t) - L_{cj}(s) Q^j_c(s) c^{*j}(t) = L_{cj}(s) q_{cj}(s) h_{ci}(t) \]
Residual Generation for Fault Isolation

\[ R_{c_i}(s) \& R_{c_j}(s) \] are arbitrary polynomials with all the roots with negative real part

- **In a similar way, output sensor isolation**
- **All the input sensors & the remaining output sensors are fault–free**

\[ \Rightarrow \text{a bank of residual generator filters is used} \]
Output Sensor Fault Isolation

- The number of these generators is equal to the number \( m \) of system outputs

- The \( i \)-th device \( (i = 1, \ldots, m) \) is driven by all but the \( i \)-th output & by all the inputs of the system

  ⇒ A fault on the \( i \)-th output sensor affects all but the \( i \)-th residual generator

  ⇒ \( y^{*i}(t) \) represents the \( m - 1 \) dimensional vector obtained by deleting from \( y^*(t) \) the \( i \)-th component

\[
y^*(t) = y(t) + f_{oi}(t)
\]

⇒ with \( f_{oi}(t) = [0 \ldots 0 h_{oi}(t) 0 \ldots 0]^T \)

\[
P(s) y(t) = Q_c(s) c(t) + Q_d(s) d(t) - p_i(s) h_{oi}(t)
\]

⇒ where \( p_i(s) \) represents the \( i \)-th column of the matrix \( P(s) \)

⇒ \( y^{*i}(t) = y^i(t) \) when a fault on the \( i \)-th output sensor \( h_{oi}(t) \) is considered
Output Sensor Fault Isolation

⇒ By multiplying by the matrix $L_{o_i}(s)$

⇒ $L_{o_i}(s)$ is a row vector belonging to the basis for the left null space of the matrix $[Q_d(s) \mid p_i(s)]$

⇒ $P^i(s)$ the matrix obtained by deleting from $P(s)$ the $i$–th column

The equation of the $i$–th filter becomes:

$$R_{o_i}(s) r_{o_i}(t) = L_{o_i}(s) P^i(s) y^*_i(t) - L_{o_i}(s) Q_c(s) c(t) = 0,$$

while, for the $j$–th filter, with $j \neq i$:

$$R_{o_j}(s) r_{o_j}(t) = L_{o_j}(s) P^j(s) y^*_j(t) - L_{o_j}(s) Q_c(s) c(t) = -L_{o_j}(s) p_i(s) h_{o_i}(t).$$

⇒ $R_{o_i}(s) \& R_{o_j}(s)$ are arbitrary polynomials whose roots have negative real part
**Sensor Fault Isolation Summary**

- Summary of the FDI capabilities of the presented schemes
- "Fault Signatures" in case of a single fault in each input & output sensor
- The residuals which are affected by faults are marked with the presence of ‘1’ in the correspondent table entry
- An entry ‘0’ means that the fault does not affect the correspondent residual

---

### Sensor Fault Isolation Table

**Fault signatures**

<table>
<thead>
<tr>
<th>Residual / Fault</th>
<th>$f_{c_1}$</th>
<th>$f_{c_2}$</th>
<th>...</th>
<th>$f_{c_{\ell_c}}$</th>
<th>$f_{o_1}$</th>
<th>$f_{o_2}$</th>
<th>...</th>
<th>$f_{o_m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{c_1}$</td>
<td>0</td>
<td>1</td>
<td>...</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>$r_{c_2}$</td>
<td>1</td>
<td>0</td>
<td>...</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>$r_{c_{\ell_c}}$</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>$r_{o_1}$</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>$r_{o_2}$</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>$r_{o_m}$</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>...</td>
<td>0</td>
</tr>
</tbody>
</table>
Sensor Fault Isolation Conditions

- When not all the elements out of the main diagonal of the table are ’1’s
  ⇒ the fault isolation is still feasible if the columns of the fault signature table are all different from each other

- When \( m - (\ell_d + 1) > 1 \)
  ⇒ all the bases of the left null space of the matrices \([Q_d(s) \mid q_{ci}(s)]\) & \([Q_d(s) \mid p_{i}(s)]\) have dimension bigger than 1
  ⇒ the degrees of freedom in the choice of the vectors \( L_{ci}(s) \) & \( L_{oi}(s) \)
   belonging to the left null space can be exploited

⇒ All the elements out of the main diagonal on the Table are ’1’s when:

- For \( i = 1, \ldots, \ell_c \), the column vectors of the matrix \( Q^i_c(s) \) & the column vectors of the matrix \( P(s) \) are not orthogonal with the row vector \( L_{ci}(s) \).

- For \( j = 1, \ldots, m \), the column vectors of the matrix \( P^j(s) \) & the column vectors of the matrix \( Q_c(s) \) are not orthogonal with the row vector \( L_{oi}(s) \).
Application Examples

1. **P(z) & Q(z) matrices are known**
   \[ R(z) r(t) = S_y(z) y(t) + S_c(z) c(t) \]

2. **P(z) & Q(z) matrices are known + measurement noise**
   \[ R(z) r(t) = S_y(z) y(t) + S_c(z) c(t) \]
   computation & sensitivity analysis

3. **\{u(t), y(t)\} multivariable non–linear process \(\leftrightarrow\) P(s) & Q(s) from linearisation**
   \[ R(s) r(t) = S_y(s) y(t) + S_c(s) c(t) \]
   computation & \(R(s)\) optimisation

---

Fault Detection in an Industrial Process

1. super heater (radiation);
2. super heater (convection);
3. super heater;
4. reheater;
5. dampers;
6. condenser;
7. drum;
8. water pump;
9. burner.
Process Description

- **120MW power plant of Pont-sur-Sambre**
  - Double-shaft industrial gas turbine
  - working in parallel with the electrical mains
- **3 major components:**
  - the reactor, turbine, & condenser

\[
\begin{align*}
    u_1(t) &: C_b & \text{gas flow} \\
    u_2(t) &: O_s & \text{turbine valves opening} \\
    u_3(t) &: Q_d & \text{super heater spray flow} \\
    u_4(t) &: R_y & \text{gas dampers} \\
    u_5(t) &: Q_a & \text{air flow} \\
    y_1(t) &: P_v & \text{steam pressure} \\
    y_2(t) &: T_s & \text{main steam temperature} \\
    y_3(t) &: T_{rs} & \text{reheat steam temperature}
\end{align*}
\]
Process Matrices: $\tilde{P}(z)$ & $\tilde{Q}(z)$

$$\tilde{P}(z) = \begin{bmatrix}
  z^4 - 1.0774z^3 - 0.1846z^2 + 0.1004z + 0.1884 & -0.1662z^2 + 0.1932z - 0.0250 & -0.1239z + 0.1086 \\
  -0.1021z^3 + 0.2958z^2 - 0.3528z + 0.1710 & z^3 - 1.6750z^2 + 0.8292z - 0.0852 & -0.0180z + 0.0156 \\
  0.1096z^2 - 0.0812z + 0.0142 & -0.2411z^2 + 0.5470z - 0.3544 & z^2 - 1.5875z + 0.6624
\end{bmatrix},$$

$$\tilde{Q}(z) = \begin{bmatrix}
  0.02 + 0.06z + 0.08z^2 - 0.1z^3 - 0.1z^4 & 0.02 + 0.06z - 0.1z^2 + 0.009z^3 & 0.05 - 0.05z - 0.03z^2 \\
  0.01 - 0.02z - 0.1z^2 - 0.1z^3 + 0.2z^4 & 0.1 - 0.1z + 0.1z^2 + 0.01z^3 & -0.1 + 0.1z + 0.04z^2 \\
  0.02 - 0.005z + 0.003z^2 - 0.003z^3 - 0.02z^4 & -0.02 - 0.09z + 0.04z^2 + 0.01z^3 & 0.09 + 0.01z - 0.01z^2 \\
  -0.09 + 0.01z + 0.02z^2 + 0.08z^3 - 0.02z^4 & -0.01 + 0.01z - 0.02z^2 - 0.001z^3 & 0.01 - 0.1z + 0.01z^2 \\
  0.01 - 0.2z - 0.1z^2 + 0.9z^3 + 0.1z^4 & -0.1 - 0.3z + 0.03z^2 + 0.1z^3 & 0.04 - 0.1z - 0.2z^2
\end{bmatrix}^T,$$

$\Rightarrow$ Kronecker invariants: $\nu_1 = 4, \nu_2 = 3, \nu_3 = 2$

Power Plant Description: *Discrete–Time Model*

$$P(z) y(t) = Q(z) u(t)$$

- $z$ is the unitary advance operator
- $P(z)$ & $Q(z)$ are polynomial matrices with dimension $(m \times m)$ & $(m \times \ell)$ respectively, with $P(s)$ nonsingular.
- $u(t) \in \mathbb{R}^\ell$ & $y(t) \in \mathbb{R}^m$, the input & output vectors of the considered discrete–time multivariable system ($t = 1, 2, \ldots, N$)
- $u(t) \equiv \mathcal{Z}[u(t)](z)$ & $y(t) \equiv \mathcal{Z}[y(t)](z)$
Residual Function & 1 Disturbance De–coupling

- $u_2(t)$ represents the disturbance signal $d_1(t)$
  $\Rightarrow \tilde{Q}_d(z) = [ \tilde{Q}_2(z) ]$

- De–coupling of the input $u_2(t)$
  $\Rightarrow \tilde{Q}_c(z) = [ \tilde{Q}_1(z) \tilde{Q}_3(z) \tilde{Q}_4(z) \tilde{Q}_5(z) ]$
  $\Rightarrow \leftrightarrow$ sensitive to $u_1(t), u_3(t), u_4(t) \& u_5(t)$

$\Rightarrow n_{f1} = 4 \& n_{f2} = 5$
$\Rightarrow$ Computation of the coefficients of the polynomials of the matrices $S_c(z)$ \& $S_y(z)$
$\Rightarrow$ Null space of $\tilde{Q}_d(z) \leftrightarrow$
$\Rightarrow \mathbf{L}(z) = \begin{bmatrix} -0.029z - 0.19z^2 & -0.048z^2 \\ 0.46 + 0.62z + 0.49z^2 & -0.21z + 0.44z^2 \\ -0.11z + 0.35z^2 & 0.42 - 0.69z + 0.32z^2 \end{bmatrix}^T$
Residual Function & 1 Disturbance De-coupling: $S_c(z)$

$$S_c(z) = \left[ S_{c1}(z), S_{c3}(z), S_{c4}(z), S_{c5}(z) \right]$$

$$S_{c1}(z) = \begin{bmatrix}
-0.0046z^4 + 0.0142z^3 - 0.0086z^2 - 0.0138z + 0.0113 \\
0.0100z^5 - 0.0243z^4 - 0.0386z^3 - 1.8 \times 10^{-6}z^2 + 0.0292z + 0.0041
\end{bmatrix},$$

$$S_{c3}(z) = \begin{bmatrix}
-0.0023z^4 + 0.0030z^3 + 0.0047z^2 + 0.0016z - 0.0039 \\
0.0087z^5 + 0.0321z^4 + 0.0209z^3 - 0.0067z^2 - 0.0115z - 0.0049
\end{bmatrix},$$

$$S_{c4}(z) = \begin{bmatrix}
0.0004z^4 - 0.0049z^3 + 0.0078z^2 - 0.0042z + 0.0008 \\
-0.0006z^5 - 0.0004z^4 + 0.0062z^3 - 0.0059z^2 + 0.0002z + 0.0009
\end{bmatrix},$$

$$S_{c5}(z) = \begin{bmatrix}
-0.0060z^4 + 0.0002z^3 + 0.0124z^2 - 0.0062z - 0.0015 \\
0.0025z^5 + 0.0096z^4 + 0.0025z^3 - 0.0169z^2 - 0.0038z - 0.0019
\end{bmatrix},$$
Residual Function & 1 Disturbance De-coupling: $S_y(z)$

\[ S_y(z) = [ S_{y_1}(z), S_{y_2}(z), S_{y_3}(z) ], \]

\[ S_{y_1}(z) = \begin{bmatrix} 0.0053z^4 - 0.0545z^3 + 0.1073z^2 - 0.0843z + 0.0269 \\ -0.0601z^5 - 0.0575z^4 + 0.1645z^3 - 0.0760z^2 + 0.0419z - 0.0045 \end{bmatrix}, \]

\[ S_{y_2}(z) = \begin{bmatrix} -0.1173z^4 + 0.4365z^3 - 0.5235z^2 + 0.2431z - 0.0326 \\ 0.4615z^5 - 0.1553z^4 - 0.5843z^3 + 0.3770z^2 - 0.0025z - 0.0154 \end{bmatrix}, \]

\[ S_{y_3}(z) = \begin{bmatrix} 0.1417z^4 - 0.4147z^3 + 0.4482z^2 - 0.2097z - 0.0349 \\ -0.1692z^4 + 0.3667z^3 - 0.2578z^2 + 0.0529z + 0.0040 \end{bmatrix}. \]
Residual Function & 1 Disturbance De-coupling: $L(z)$

\[
L(z) = \begin{bmatrix}
-0.029z - 0.19z^2 & -0.048z^2 \\
0.46 + 0.62z + 0.49z^2 & -0.21z + 0.44z^2 \\
-0.11z + 0.35z^2 & 0.42 - 0.69z + 0.32z^2
\end{bmatrix}^T
\]

- It can be evaluated by means of the command `null` (Matlab Polynomial Toolbox) of the matrix $\tilde{Q}_d(z)$

Residual Function & 2 Disturbance De-coupling

- $u_2(t) \& u_4(t) \leftrightarrow$ disturbance signals $d_1(t) \& d_2(t)$
  \[\Rightarrow \tilde{Q}_d(z) = \begin{bmatrix} \tilde{Q}_2(z) & \tilde{Q}_4(z) \end{bmatrix} \leftrightarrow\]

- input $u_2(t) \& u_4(t)$ de-coupling
  \[\Rightarrow \tilde{Q}_c(z) = \begin{bmatrix} \tilde{Q}_1(z) & \tilde{Q}_3(z) & \tilde{Q}_5(z) \end{bmatrix}\]
  \[\Rightarrow \leftrightarrow$ sensitive to $u_1(t), u_3(t) \& u_5(t)$
Residual Function & 2 Disturbance De-coupling

⇒ $n_f = 9$

⇒ Computation of the coefficients of the polynomials of the matrices $S_c(z)$ & $S_y(z)$

⇒ Null space of $\tilde{Q}_d(z) \hookrightarrow$

$L(z) = \begin{bmatrix} -0.029 + 0.17z - 0.34z^2 + 0.2z^3 + 0.0016z^4 - 0.005z^5 \\ -0.024 + 0.036z - 0.092z^2 + 0.18z^3 + 0.43z^4 - 0.62z^5 + 0.042z^6 \\ -0.018 + 0.068z - 0.11z^2 + 0.11z^3 + 0.17z^4 - 0.37z^5 + 0.13z^6 + 0.016z^7 \end{bmatrix}^T$

Residual Function & 2 Disturbance De-coupling: $S_c(z)$

$S_c(z) = \left[ S_{c1}(z), S_{c2}(z), S_{c3}(z) \right]$

$S_{c1}(z) = \left[ 0.0002z^9 - 0.0077z^8 + 0.0315z^7 - 0.0257z^6 - 0.0056z^5 + 0.0067z^3 + 0.0015z^2 - 0.0022z^2 + 0.0027z - 0.0011 \right]$,

$S_{c3}(z) = \left[ 0.0003z^9 - 0.0044z^8 - 0.0092z^7 + 0.0118z^6 + 0.0074z^5 + 0.0063z^4 + 0.0016z^3 - 0.0034z^2 + 0.0015z - 0.0001 \right]$,
Residual Function & 2 Disturbance De–coupling: $S_c(z)$

$$S_c(z) = \left[ S_{c_1}(z), S_{c_2}(z), S_{c_3}(z) \right]$$

$$S_{c_5}(z) = \left[ -0.0004 z^9 - 0.0016 z^8 + 0.0028 z^7 + 0.0015 z^6 - 0.0026 z^5 +
+0.0001 z^4 + 0.0036 z^3 - 0.0048 z^2 + 0.0016 z - 3 \times 10^{-5} \right],$$

Residual Function & 2 Disturbance De–coupling: $S_y(z)$

$$S_y(z) = \left[ S_{y_1}(z), S_{y_2}(z), S_{y_3}(z) \right]$$

$$S_{y_1}(z) = \left[ -0.0031 z^9 + 0.0360 z^8 - 0.0172 z^7 - 0.0988 z^6 +
+0.1273 z^5 - 0.0550 z^4 + 0.0262 z^3 - 0.0307 z^2 + 0.0194 z - 0.0044 \right],$$

$$S_{y_2}(z) = \left[ 0.0167 z^9 - 0.3144 z^8 + 0.7071 z^7 - 0.5489 z^6 +
+0.0923 z^5 + 0.1046 z^4 - 0.1078 z^3 + 0.0702 z^2 - 0.0241 z + 0.0027 \right],$$
Residual Function & 2 Disturbance De-coupling: \( S_y(z) \)

\[
S_y(z) = [ S_{y1}(z), S_{y2}(z), S_{y3}(z) ]
\]

\[
S_{y3}(z) = [0.0124z^9 - 0.0380z^8 + 0.0272z^7 + 0.0422z^6 + \\
-0.0740z^5 - 0.0074z^4 + 0.0979z^3 - 0.0886z^2 + 0.0329z - 0.0044],
\]

Residual Function & 2 Disturbance De-coupling: \( L(z) \)

- \( n_f = 9 \) & \( L(z) \) given by:

\[
L(z) = \begin{bmatrix}
-0.029 + 0.17z - 0.34z^2 + 0.2z^3 + 0.0016z^4 - 0.005z^5 \\
-0.024 + 0.036z - 0.092z^2 + 0.18z^3 + 0.43z^4 - 0.62z^5 + 0.042z^6 \\
-0.018 + 0.068z - 0.11z^2 + 0.11z^3 + 0.17z^4 - 0.37z^5 + 0.13z^6 + 0.016z^7
\end{bmatrix}^T,
\]
Residual Function & 2 Disturbance De-coupling: $L(z)$

- Computed by the Matlab null space [Forney, 1975; Chen, 1984] of the matrix $\tilde{Q}_d(z)$
  - The Matlab function `null` computes a polynomial basis for the right null space of a polynomial matrix
- This basis is minimal in the sense of Forney [Forney, 1975; Kailath, 1980], i.e.:
  - (i) it is column reduced & has full column rank (irreducible)
  - (ii) it has minimal order

2. Residual Function $r(t)$ with Measurement Noise

- $P(z)$ & $Q(z)$ is the linear model of Pont-sur-Sambre
  - $\{u(t), y(t)\}$ simulated from the linear model + additive noise
    - $c^*(t) = c(t) + \tilde{\sigma}_c(t)$
    - $y^*(t) = y(t) + \tilde{\sigma}_y(t)$
  - Residual generator $R(z) r(t) = S_y(z) y^*(t) + S_c(z) c^*(t)$
  - from the linear model
2. Residual Function $r(t)$ with Measurement Noise

$$R(s) r(t) = \frac{L(s) P(s) y^*(t) - L(s) Q_c(s) c^*(t) + L(s) Q_f(s) f(t)}{}$$

measurement noise $\neq 0$

faulty case

$$\Rightarrow \{\tilde{\sigma}_c(t), \tilde{\sigma}_y(t)\} \leq 10\%$$

$$\Rightarrow R(z) \leftrightarrow \text{fault sensitivity maximisation}$$

$$\Rightarrow 1 \text{ or } 2 \text{ de-coupled disturbances + fault isolation } \leftrightarrow n_f = 4, 5, \& 9$$

2. Fault Detection Capabilities with Noise

<table>
<thead>
<tr>
<th>Noise</th>
<th>$f_{c1}$ %</th>
<th>$f_{c2}$ %</th>
<th>$f_{c3}$ %</th>
<th>$f_{c4}$ %</th>
<th>$f_{c5}$ %</th>
<th>$f_{y1}$ %</th>
<th>$f_{y2}$ %</th>
<th>$f_{y3}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1%</td>
<td>0.8226</td>
<td>0.32</td>
<td>0.48</td>
<td>0.19</td>
<td>0.47</td>
<td>0.61</td>
<td>0.01</td>
<td>0.50</td>
</tr>
<tr>
<td>1 %</td>
<td>3.72</td>
<td>0.90</td>
<td>2.16</td>
<td>0.60</td>
<td>1.12</td>
<td>1.83</td>
<td>0.91</td>
<td>3.31</td>
</tr>
<tr>
<td>10 %</td>
<td>7.44</td>
<td>2.32</td>
<td>4.36</td>
<td>1.20</td>
<td>4.53</td>
<td>15.86</td>
<td>5.47</td>
<td>28.75</td>
</tr>
<tr>
<td>20 %</td>
<td>9.92</td>
<td>2.54</td>
<td>7.27</td>
<td>2.40</td>
<td>6.80</td>
<td>31.12</td>
<td>9.13</td>
<td>63.24</td>
</tr>
<tr>
<td>30 %</td>
<td>14.88</td>
<td>3.51</td>
<td>10.18</td>
<td>3.60</td>
<td>9.07</td>
<td>47.60</td>
<td>12.78</td>
<td>94.55</td>
</tr>
<tr>
<td>40 %</td>
<td>17.36</td>
<td>4.68</td>
<td>11.63</td>
<td>4.80</td>
<td>11.29</td>
<td>54.31</td>
<td>15.52</td>
<td>110.52</td>
</tr>
</tbody>
</table>

$$\Rightarrow \text{Minimal detectable fault}$$

$$\Rightarrow \text{Increasing additive noise amplitude}$$
2. Fault Sensitivity with Measurement Noise

Variable Measurement Noise

First & second monitored fault-free residuals for the case of one disturbance signal ($\ell_d = 1$).
2. Fault–Free Residual Signals (2 disturbance signals)

Monitored fault–free residual for the case of 2 disturbance signals ($\ell_d = 2$).

2. Fault–Free & Faulty Residual Signals

Faults on the first input & first output when 1 disturbance ($\ell_d = 1$) has been de–coupled.
3. Residual Function $r(t)$ with Model Uncertainty

- $\{P(s), Q(s)\}$ model linearisation of a Piper PA–30 aircraft non–linear system
  - $\Rightarrow \{u(t), y(t)\}$ & faults simulated from the non–linear model simulator
  - $\Rightarrow$ Residual generator $R(s) r(t) = S_y(s) y(t) + S_c(s) c(t)$ from the linearised model
  - $\Rightarrow R(s)$ poles by trial & error procedure $\Leftarrow$ fault sensitivity maximisation
  - $\Rightarrow 3 + 1$ de–coupled disturbances $\Leftarrow$ wind gust + fault isolation;
    $n_f = 3, \ldots, 5$
Piper PA–30 Model Nomenclature

- $V$: True Air Speed (TAS)
- $\delta_e$: elevator deflection angle
- $\alpha$: angle of attack
- $\delta_a$: aileron deflection angle
- $\beta$: angle of sideslip
- $\delta_r$: rudder deflection angle
- $P$: roll rate
- $\delta_{th}$: throttle aperture percentage
- $Q$: pitch rate
- $R$: yaw rate
- $\phi$: bank angle
- $\theta$: elevation angle
- $\psi$: heading angle
- $H$: altitude
- $\gamma$: flight path angle
- $n$: engine r.p.m.
- $m$: airplane mass
- $I_{x}$, $I_{y}$, $I_{z}$: airplane inertia moments matrix
- $F_x$, $F_y$, $F_z$: total force components along body axes
- $M_x$, $M_y$, $M_z$: total moment components along body axes

\[
\begin{align*}
\dot{V} &= \frac{F_x \cos \alpha \cos \beta}{m} + \frac{F_y \sin \beta}{m} + \frac{F_z \sin \alpha \cos \beta}{m} \\
\dot{\alpha} &= -\frac{F_x \sin \alpha + F_z \cos \alpha}{mV \cos \beta} + \frac{Q - (P \cos \alpha + R \sin \alpha) \tan \beta}{m} \\
\dot{\beta} &= -\frac{F_x \cos \alpha \sin \beta + F_y \cos \beta - F_z \sin \alpha \sin \beta}{mV} + \frac{P \sin \alpha + R \cos \alpha}{mV} \\
\dot{P} &= \frac{M_x I_z + M_z I_{xz} + P Q I_{x} (I_x - I_y + I_z)}{I_x I_z - I_{xz}^2} + \frac{Q R (I_y I_z - I_{xz}^2 - I_{zz}^2)}{I_x I_z - I_{xz}^2}
\end{align*}
\]
Piper PA–30 Model (2)

\[
\begin{align*}
\dot{Q} &= \frac{M_y + PR(I_z - I_x) - P^2 I_{xz} + R^2 I_{xx}}{I_y} \\
\dot{R} &= \frac{M_x I_{xz} + M_z I_x + PQ(I_x^2 - I_x I_y + I_{xz}^2)}{I_x I_z - I_{xz}^2} + \\
&\quad + \frac{QR I_{xz} (I_x + I_y - I_z)}{I_x I_z - I_{xz}^2} \\
\dot{\phi} &= P + Q \sin \phi \tan \theta + R \cos \phi \tan \theta \\
\dot{\theta} &= Q \cos \phi - R \sin \phi 
\end{align*}
\]

Input–Output Canonical Form Computation

\[
\begin{align*}
\hat{p}(s) & & \hat{q}(s) \text{ from model linearisation:} \\
\dot{x}(t) &= Ax(t) + Bc(t) + Ed(t) \text{ with} \\
x(t) &= [\Delta V \ \Delta \alpha \ \Delta \beta \ \Delta P \ \Delta Q \ \Delta R \ \Delta \phi \ \Delta \psi \ \Delta H \ \Delta n]^T \\
c(t) &= [\Delta \delta_e \ \Delta \delta_a \ \Delta \delta_r \ \Delta \delta_{th}]^T \\
d(t) &= [w_u \ w_v \ w_w]^T \\
y(t) &= [\Delta V \ \Delta P \ \Delta Q \ \Delta R \ \Delta \phi \ \Delta \theta \ \Delta \psi \ \Delta H \ \Delta n]^T 
\end{align*}
\]
Residual Function $r(t)$ with Model Uncertainty

- $u(t) \in \mathbb{R}^4$, $y(t) \in \mathbb{R}^9$ obtained from a non-linear Simulink model of Piper PA30
- With $\tilde{P}(s)$ & $\tilde{Q}(s)$ derived from model linearisation
  \[ R(s) r(t) = L(s) \tilde{P}(s) y(t) - L(s) \tilde{Q}_c(s) c(t) + L(s) \tilde{Q}_f(s) f(t) \]
  \[ L(s) \tilde{P}(s) y(t) - L(s) \tilde{Q}_c(s) c(t) \] is model mismatch error
- $R(s)$ & $L(s)$: polynomial matrices optimisation $\hookrightarrow$ GAOT for Matlab
  \[ \ell_d = 3 \text{ (wind gusts)} + 1 \text{ (input or output)} \]

Scheme for Input Sensor FDI
Faulty $\{u(t), y(t)\}$ simulated by the non–linear model

$\Rightarrow$ Wind gusts: $(w_u, w_v, w_w)$. Correlation times & variances: $\tau_u = 2.326[s]$, $\tau_v = 7.143[s]$, $\tau_w = 0.943[s]$, $E[w_u^2] = E[w_v^2] = E[w_w^2] = 0.7[(m/s)^2]$.

$\Rightarrow$ Trajectory:
- radius of curvature $1000[m]$
- speed $V = 50[m/s]$
- altitude $H = 330[m]$
- flap $= 0^\circ$.

$\Rightarrow$ Detailed model of Inertial Measurement Unit (MEMS technology), Air Data Computer, Heading Reference System.
Simulator Block Brief Description (1)

Command Surfaces Deflection Measurements

⇒ \( \delta_e, \delta_a, \delta_r, \delta_{th} \) acquired with a sample rate of 100Hz by means of potentiometers

⇒ Errors modelled by bias & white noise:

<table>
<thead>
<tr>
<th>Input sensor</th>
<th>Bias</th>
<th>White Noise Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevator deflection angle</td>
<td>0.0052 rad</td>
<td>0.0053 rad</td>
</tr>
<tr>
<td>Aileron deflection angle</td>
<td>0.0052 rad</td>
<td>0.0053 rad</td>
</tr>
<tr>
<td>Rudder deflection angle</td>
<td>0.0052 rad</td>
<td>0.0053 rad</td>
</tr>
<tr>
<td>Throttle aperture</td>
<td>1%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Angular Rate Measurement

⇒ Angular rate measures by 3 gyroscopes of (IMU) with sample rate of 100Hz

⇒ Measurement errors:

- Non unitary scale factor: multiplicative factor \( \in [0.99, 1.01] \).
- Alignment error of spin axes with respect to body (reference) axes: six error angles up to 1 deg (uniform random variables)
- Limited bandwidth of the considered gyro (10 Hz).
- \( g \)-sensitivity \( 72 \, \text{deg/h} \).
- Additive white noise (216 deg/h).
- Gyro drift, coloured stochastic process, 1080 deg/h std. dev. & a decay time of 20 min.
Simulator Block Brief Description (3)

- **Attitude Angle Measurement**
  
  ⇒ angles generated by a digital filtering system based on a DSP that processes both the angular rate & the accelerations provided by the IMU with a sample rate of 100Hz
  
  ⇒ 2 Measurement errors correlated by a first order filter system with time constant equal to 60 sec

  - A systematic error generated by the apparent vertical.
  - A white noise modelling the imperfection of both the system & the environment influences.

  ⇒ the resulting attitude angle measurements are affected by an additive coloured noise with std.dev. of 1 deg

Simulator Block Brief Description (4a)

- **Air Data System (ADS)**
  
  ⇒ the ADS unit consists of an Air Data Computer (ADC) providing measures with a sample rate of 1 Hz
  
  ⇒ Errors affecting the TAS:

  - Calibration error affecting the differential pressure sensor. This error leads to a TAS computation systematic error, performed the ADC, fulfilling the ARINC (Aeronautical Radio Inc.) accuracy requirements (2 m/sec).
  - Additive coloured noise due to wind gusts (std. dev. 1 & correlation time 2.3 sec).
  - Additive white noise (std. dev. 0.5 m/sec) modelling the imperfection of the system & the environment influences.
Simulator Block Brief Description (4b)

- **Air Data System (ADS)**

  ⇒ Altitude errors:
  
  - Calibration error affecting the static pressure sensor. This error leads to an altitude computation systematic error, performed the ADC, fulfilling the ARINC accuracy requirements (5 m).
  - Additive White noise (std. dev. 1 m) modelling the imperfection of the system & the environment influences.

Simulator Block Brief Description (5)

- **Heading Reference System (HRS)**

  ⇒ Magnetic compass coupled to a directional gyro

  ⇒ Measurement errors correlated by a first order filter with time constant equal to 60 sec:

  - a systematic error generated by a bias of the magnetic compass (1 deg),
  - a white noise modelling the imperfection of the system & the environment influences.

  ⇒ The resulting heading measurement is affected by an additive coloured noise std.dev. 1 deg
Simulator Block Brief Description (6)

- **Engine Shaft Rate Measurement**
  ⇒ Incremental encoder white noise. Quantisation error resolution of 10000 pulse/rev

- **Servo Actuator Models**
  ⇒ second order linear models with saturations

- **Dryden Atmosphere Model**
  ⇒ Model block of the Aerospace Blockset of Matlab® 6.5
  ⇒ Turbulence obtained by forming filter excited by a band–limited white noise (U.S. Military Specification MIL–F–8785C)

FDI Filter Design

- Filters fed by 4 component $c(t)$ & by 9 component $y(t)$

- The input & output sequences are affected by the measurement errors

- 4 residual generator filter bank used to detect input sensor for the 4 control variables
  
  \[ c(t) = [\Delta \delta_e(t), \Delta \delta_a(t), \Delta \delta_r(t), \Delta \delta_{th}(t)]^T \]

- Fault *isolation* properties if each residual generator is fed by all but one the 4 inputs & by the 9 outputs $y(t) = [\Delta V(t), \Delta P(t), \Delta Q(t), \Delta R(t), \Delta \phi(t), \Delta \theta(t), \Delta \psi(t), \Delta H(t), \Delta n(t)]^T$

- The output variables $\Delta \alpha(t)$ & $\Delta \beta(t)$ not considered as critical to obtain
FDI Filter Design: De–coupling Issues

- Each filter bank is independent of one of the 4 inputs
- insensitive to the corresponding fault signals
- Residual generator bank is be de–coupled from 3 wind gusts \( d(t) = [w_u(t), w_v(t), w_w(t)]^T \)
- FDI capability related to the properties of the residual generators with measurement errors, modelling approximations & un–decoupled disturbance signals
- Filter robustness properties in terms of fault sensitivity & disturbance insensitivity

FDI Filter Design: Optimisation

- Filter synthesis performed by choosing residual generator linear combination of residual generators maximising the steady–state gain of the transfer functions between input sensor fault signals \( f_{ci}(t) \) & residual functions \( r_{cj}(t) \) \((i, j = 1, \ldots, 4, j \neq i)\)
- Polynomial roots \( R_{cj}(s) \) optimised numerically [GAOT, 1995] for obtaining suitable transient dynamics.
- Aircraft operating conditions with different faults were simulated
- Faults in single input–output sensors are generated by in the input–output signals \( c(t) \) & \( y(t) \).
FDI Filter Remarks: Fault Detection

- The residual signals indicate fault occurrence if their values are lower or higher than the fault–free thresholds.

- The thresholds depend on the residual errors due to measurement errors, linearised model approximations & un–decoupled disturbance signals.

- Positive & negative threshold 10% margins on the maximum & minimum values of the fault–free residual signals.

Examples of Faulty & Fault–Free Residuals

Bank residuals for the 1st input sensor fault $f_{c1}(t)$ isolation.
FDI Filter Design Remarks

- The first residual for the input sensor $f_{c_1}(t)$ independent of a fault on the input sensor itself: $r_{c_1}(t)$ filter designed to be sensitive to the input signal $c_1^*(t)$

- Filter parameter optimisation, i.e. the the roots $-1/\tau_i (i = 1, 2, \ldots, n_f)$ & of real constants $k_i$, obtained by means of the Genetic Algorithm Optimisation Toolbox (GAOT) [GAOT, 1995] for Matlab® (local minima problems)

- $R_{c_i}(s)$ polynomial matrix roots optimised & placed between $-1$ & $-10^{-2}$ for maximising fault detection promptness & for minimising false alarm rate

Examples of Faulty & Fault–Free Residuals

Residuals of the bank for the isolation of the $9^{th}$ output sensor fault $f_{o_9}(t)$. 
FDI Filter Design Remarks

- Minimal fault detection capabilities obtained by optimising maximum & minimum values of $r_{c_i}(t)$ & $r_{o_j}(t)$ in fault-free conditions (acceptable false-alarms rate)

- Minimal detectable step faults on the various sensors are collected in the Tables

- First Table collects the minimal detectable step fault simulated on the input sensors: $r_{c_j}(t)$ monitored for FDI of the considered input fault case $f_{c_i}(t)$ ($i, j = 1, \ldots, 4, i \neq j$)

- Second Table collects minimal detectable step fault amplitudes on output sensors: $r_{o_j}(t)$ monitored for FDI of output sensor fault case $f_{o_i}(t)$ ($i, j = 1, \ldots, 9, i \neq j$)

### Minimal Detectable Input Faults

Minimal detectable step input sensor faults.

<table>
<thead>
<tr>
<th>Input Sensor Variable $c_i(t)$</th>
<th>Fault Size</th>
<th>Detection Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevator deflection angle</td>
<td>2°</td>
<td>18 sec</td>
</tr>
<tr>
<td>Aileron deflection angle</td>
<td>3°</td>
<td>6 sec</td>
</tr>
<tr>
<td>Rudder deflection angle</td>
<td>4°</td>
<td>8 sec</td>
</tr>
<tr>
<td>Throttle aperture %</td>
<td>2%</td>
<td>15 sec</td>
</tr>
</tbody>
</table>
Minimal detectable step output sensor faults.

<table>
<thead>
<tr>
<th>Output Sensor Variable $y_i(t)$</th>
<th>Fault Size</th>
<th>Detection Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Air Speed</td>
<td>8 m/sec</td>
<td>27 sec</td>
</tr>
<tr>
<td>Pitch Rate</td>
<td>3 deg/sec</td>
<td>22 sec</td>
</tr>
<tr>
<td>Elevation Angle</td>
<td>5 deg</td>
<td>28 sec</td>
</tr>
<tr>
<td>Altitude</td>
<td>8 m</td>
<td>12 sec</td>
</tr>
<tr>
<td>Roll Rate</td>
<td>2 deg/sec</td>
<td>24 sec</td>
</tr>
<tr>
<td>Yaw Rate</td>
<td>3 deg/sec</td>
<td>29 sec</td>
</tr>
<tr>
<td>Bank Angle</td>
<td>5 deg</td>
<td>5 sec</td>
</tr>
<tr>
<td>Heading Angle</td>
<td>6 deg</td>
<td>25 sec</td>
</tr>
<tr>
<td>Engine Angular Rate</td>
<td>20 RPM</td>
<td>30 sec</td>
</tr>
</tbody>
</table>

- The minimal detectable fault values in Tables expressed in the unit of measure of sensor signals & s.t. any fault occurrence must be detected & isolated asap
- Detection delay time evaluated & based on the slowest residual crossing time w.r.t. the settled thresholds
- Residual generator performance seems to assess the diagnostic capabilities of the technique
- FDI strategy appears promising for diagnostic application to general aviation aircrafts
- Similar results obtained by dynamic observers, UIO or Kalman filters: but the corresponding realisations require a more complex design & an higher cost implementation
Residual Function Examples: faulty & fault–free residuals

Aileron actuator fault $\delta_{a}$

Rudder actuator fault $\delta_{r}$.

Final Remarks

- Some results are shown in FDI of sensor faults of dynamic system by using a model–based approach

- Different types of fault having a barely detectable effect on anyone measurement, can be detected easily using a bank of residual generator in the form of dynamic filters

- An important aspect of the approach suggested here to FDI is the simplicity of structure of the technique used to generate the residual functions for FDI, when compared with traditional schemes e.g. based on banks of Unknown Input Observers (UIO) & Kalman filters.
Final Remarks

- The method outlined focuses to some extent on input–output or state–space concepts; the actual algorithm is based only on input–output processing of all measurable signals ($c(t)$ & $y(t)$)

- Algorithmic simplicity important when verification & validation of a demonstrable scheme for air–worthiness certification needed

- Complex computations & schemes require high cost & complexity w.r.t. certification

- Modelling uncertainty & measurement noise well tackled: this method used in real applications

- Further studies for evaluating effectiveness on real aircraft system data

References: General FDI


References: **Same Author**


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References: **Related Works**

References: Canonical Forms


References: Algorithms