Nonlinear (Fault Diagnosis and) Fault Tolerant Control Schemes for Aerospace Applications

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Main Topics

1. **Differential geometry fundamentals**
2. **The NonLinear Geometric Approach (NLGA) and its application to Fault Detection and Isolation (FDI) and Fault Detection and Diagnosis (FDD) for NonLinear Input Affine Systems (NLIAS)**
3. **FDI and FDD for a general aviation aircraft : residual generators wind decoupled. Test on a Piper PA 30 detailed flight simulator.**
4. **Active Fault Tolerant Control Schemes (AFTCS) for general aviation aircrafts and UAVs:**
   - **AFTCS: from FDI module to FDD module**
   - **A novel AFTCS scheme developed by means of an FDD module based on NonLinear Geometric Approach - Adaptive Filters (NLGA-AF)**
   - **FDD module and Guidance and Control System (GCS) can be independently designed**
   - **Case studies: 6 Dof model**
     - **used for a general aviation aircraft, can be approximate by NonLinear Input Affine System (NLIAS)**

Main topics continued

5. **Longitudinal guidance and control system, for an aircraft affected by wind shear disturbances**

   - **Wind shear components regarded as fault to be estimated**
   - **Wind Shear estimate exploited for BackStepping based control signals**, thus **resulting in an Adaptive BackStepping Control Scheme**
   - **Thanks to good wind shear estimates the controller can thus correct the aircraft behavior in real-time**
   - **An accurate aircraft model and wind shear data from a simulated actual crash situation showed the effectiveness of the overall guidance and control system in wind worst case conditions and also in presence of model uncertainties**

Differential Geometry: Fundamentals

The notion of Lie-derivative and Lie-product play central role in the geometric approach of nonlinear system analysis and control.

Lie-derivative

Let us given a nonlinear scalar-valued function $\lambda : \mathbb{R}^n \to \mathbb{R}$

and a **vector field** $f : \mathbb{R}^n \to \mathbb{R}^n$ on a common domain $U = dom(\lambda) = dom(f) \subseteq \mathbb{R}^n$

open and let λ be continuously differentiable, i.e. $\lambda \in C^1$ on U

The derivative of λ along f is defined as

$$\boxed{L_f\lambda(x) = \frac{\partial \lambda(x)}{\partial x} f(x) = \sum_{i=1}^{n} \frac{\partial \lambda(x)}{\partial x_i} f_i(x) = \langle d\lambda(x), f(x) \rangle}$$

Observe that the vector field f in any point defines a direction along which we compute the derivative (gradient) of λ

Differential Geometry Fundamentals and Tools

continued

It is important to note that $L_f\lambda\left(x\right) : \mathbb{R}^n \to \mathbb{R}$.

The repeated Lie derivation is defined recursively $L_f^k\lambda\left(x\right) = \dfrac{\partial\left(L_f^{k-1}\lambda\left(x\right)\right)}{\partial x} f(x)$

with $L_f^0\lambda\left(x\right) = \lambda\left(x\right)$

Example

$$\lambda\left(x_1, x_2\right) = x_1^2;\; f\left(x_1, x_2\right) = \begin{bmatrix} x_1^2 + x_2^2 \\ x_1^3 + x_2^3 \end{bmatrix} \Rightarrow L_f\left(x_1, x_2\right) = 2x_1\left(x_1^2 + x_2^2\right)$$

Differential Geometry Fundamentals and Tools

continued

Lie-Product: **is defined for two vector fields** $f, g : \mathbb{R}^n \rightarrow \mathbb{R}^n$ defined on a joint open domain $U = dom(f) = dom(g) \subseteq \mathbb{R}^n$ where both f, g are continuous and differentiable (C^1 on U):

$$\boxed{[f,g](x) = \frac{\partial g(x)}{\partial x} f(x) - \frac{\partial f(x)}{\partial x} g(x)}$$

Lie Product (or Lie Bracket)

where

$$\frac{\partial g(x)}{\partial x} = \begin{bmatrix} \frac{\partial g_1(x)}{\partial x_1} & \frac{\partial g_1(x)}{\partial x_2} & \cdots & \frac{\partial g_1(x)}{\partial x_n} \\ \frac{\partial g_2(x)}{\partial x_1} & \frac{\partial g_2(x)}{\partial x_2} & \cdots & \frac{\partial g_n(x)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial g_n(x)}{\partial x_1} & \frac{\partial g_n(x)}{\partial x_2} & \cdots & \frac{\partial g_n(x)}{\partial x_n} \end{bmatrix}$$

Note:

Naturally $[f,g](x) \in \mathbb{R}^n$ **which enables to apply the Lie-bracket operation repeatedly.**

Differential Geometry Fundamentals and Tools

continued

Lie-Product properties :

Bilinearity over $\mathbb{R}$:

$$f_1, f_2, g_1, g_2 : \mathbb{R}^n \rightarrow \mathbb{R}^n; r_1, r_2 \in \mathbb{R}$$
$$\left[r_1 f_1 + r_2 f_2, g_1 \right](x) = r_1 \left[f_1, g_1 \right] + r_2 \left[f_2, g_1 \right]$$
$$\left[f_1, r_1 g_1 + r_2 g_2 \right](x) = r_1 \left[f_1, g_1 \right] + r_2 \left[f_1, g_2 \right]$$

Skew commutativity:

$$f, g : \mathbb{R}^2 \rightarrow \mathbb{R}^n$$
$$\left[f, g \right] = -\left[g, f \right]$$

Jacobi identity:

$$\left[f \quad \left[g \quad p \right] \right] + \left[g \quad \left[p \quad f \right] \right] + \left[p \quad \left[f \quad g \right] \right] = 0$$

Differential Geometry Fundamentals and Tools

continued

- **Example:** The following simple two-dimensional ($x \in \mathbb{R}^2$) examples illustrate the definition of Lie-products.

1) Two general vector field

$$f(x_1, x_2) = \begin{bmatrix} x_1^2 + x_2^2 \\ x_1^3 + x_2^3 \end{bmatrix}; g(x) = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$$

$$[f, g](x) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1^2 + x_2^2 \\ x_1^3 + x_2^3 \end{bmatrix} - \begin{bmatrix} 2x_1 & 2x_2 \\ 3x_1^2 & 3x_2^2 \end{bmatrix} \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$$

2) A constant vector field with a general vector field

$$\bar{f}(x) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}; g(x) = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}$$

$$[\bar{f}, g](x) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

Differential Geometry Fundamentals and Tools

continued

3) Two constant vector fields

$$\bar{f}(x) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}; \, g(x) = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$[\bar{f}, g](x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Differential Geometry Fundamentals and Tools continued

Distributions, Co-distributions

Distributions and co-distributions are important basic notions in the geometric approach to analyzing dynamic properties of nonlinear systems, such as controllability and observability.

Consider a set of functions $f_1, \cdots, f_d : \mathbb{R}^n \rightarrow \mathbb{R}^n$ with joint domain U in $\mathbb{R}^n$

$$U = dom(f_1) = \cdots = dom(f_d) \subseteq \mathbb{R}^n$$

The distribution Δ **spanned by the functions** $f_1, \cdots, f_d$

$$\boxed{\Delta(x) = span(f_1(x), \cdots, f_d(x))}$$

assigns a vector space to each point $x \in U$.

In the preceding definition span is understood to mean

$\alpha_1(x) f_1(x) + \cdots + \alpha_d(x) f_d(x)$ **with** $\alpha_i(x)$ **all smooth functions of** x

Differential Geometry Fundamentals and Tools

continued

Remark

A distribution is a smooth assignment of a subspace of $\mathbb{R}^n$ to each point x

Dimension of a distribution

The dimension of a distribution Δ at x is the dimension of the vector space spanned by Δ in x.

Operations on Distributions. The following set-like operations are defined on distributions:

$$
\begin{aligned}
(\Delta_1 + \Delta_2)(x) &= \Delta_1(x) + \Delta_2(x) \\
(\Delta_1 \cap \Delta_2)(x) &= \Delta_1(x) \cap \Delta_2(x)
\end{aligned}
$$

Differential Geometry Fundamentals and Tools

continued

Obviously

$$\Delta_1 = span\left[f_1 \quad \cdots \quad f_n\right]; \Delta_2 = span\left[g_1 \quad \cdots \quad g_m\right]$$
$$\Rightarrow \Delta_1 + \Delta_2 = span\left[f_1 \quad \cdots \quad f_n, g_1 \quad \cdots \quad g_m\right]$$

Properties of distributions

1) Nonsingular distribution Δ on U : Δ is a non singular distribution if $\exists d \in N$ such that $\dim\left(\Delta\left(x\right)\right) = d \quad \forall x \in U$.

2) x_0 **is a regular point of a distribution** Δ , if there exists a neighborhood U^0 of x_0 such that Δ is **not singular** in x_0

3) A point of singularity is not a regular point

4) f belongs to the distribution Δ when $f\left(x\right) \in \Delta\left(x\right), \forall x$

5) Distribution Δ_1 contains Δ_2 ($\Delta_1 \supset \Delta_2$) if $\Delta_1\left(x\right) \supset \Delta_2\left(x\right), \quad \forall x$

Differential Geometry Fundamentals and Tools

continued

Distributions involutive

A distribution Δ is involutive if $\boxed{\tau_1, \tau_2 \in \Delta \Rightarrow [\tau_1, \tau_2] \in \Delta}$

Distribution is invariant under the vector field f when $\tau \in \Delta \Rightarrow [f, \tau] \in \Delta$

Example

Consider the set of function $f_i : \mathbb{R}^n \to \mathbb{R}^n$ defined as

$$f_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ x_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

where only the i entry it is non zero $i = 1, \cdots, n$. **The dimension of the distribution** $\Delta(x) = span\left[f_1 \quad \cdots \quad f_n \right]$ **depend on point** x **as follows:**

- $\dim(\Delta(x)) = n$ when $x_i \neq 0, i = 1, \cdots, n$
- $\dim(\Delta(x)) = d < n$ if $\exists i : x_i = 0$
- $\dim(\Delta(x)) = 0$ when $x_i = 0, i = 1, \cdots, n$

Therefore $\Delta(x)$ is non singular everywhere except in the set $D_{\text{sing}} = \{ x \mid \exists i, x_i = 0, i = 1, \cdots, n \}$

Differential Geometry Fundamentals and Tools

continued

Co-Distributions

Co-distributions are defined using the notions of dual space and co-vector fields which are as follows.

Dual Space of a Vector Space

Dual space V^* of a vector space $V \subset \mathbb{R}^n$ is the set of all linear real-valued functions defined on V. Formally defined as

$$f(x) = f\begin{pmatrix} x_1 & \cdots & x_n \end{pmatrix} = a_1 x_1 + \cdots a_n x_n, \alpha_i \in \mathbb{R}, i = 1, \cdots, n$$

i.e. $f(x) = ax, a = [a_1, \cdots, a_n]; x = [x_1, \cdots, x_n]^T$

Note that f is given by the row vector a

Co-Vector Field

A mapping from $\mathbb{R}^{n \times 1}$ to $\mathbb{R}^{1 \times n}$ is called a co-vector field. It can be represented row vector valued function

$$f(x) = f(x_1, \cdots, x_n) = [f_1(x), \cdots, f_n(x)]$$

Differential Geometry Fundamentals and Tools

continued

Example

Let us define a co-vector field $d\lambda$ associated to a vector field $\lambda : \mathbb{R}^n \to \mathbb{R}$

As follows $d\lambda(x) = \begin{bmatrix} \dfrac{\partial \lambda}{\partial x_1} & \cdots & \dfrac{\partial \lambda}{\partial x_n} \end{bmatrix}$ is the gradient of λ

Co-Distributions

Let $\omega_1, \cdots, \omega_n$ be smooth co-vector fields. Then Ω is a co-distribution spanned by the co-vectors: $\Omega = span\left\{ \omega_1 \quad \cdots \quad \omega_d \right\}$

At any point x_0 co-distributions are subspaces of $\left(\mathbb{R}^n\right)^*$

Operations on Co-distributions and their Properties.

Operations, such as addition, intersection, inclusion are defined in an analogue way to that of distributions. Likewise, the notion of the dimension of a codistribution at a point, regular point, point of singularity are applied to co-distributions in an analogous way.

Differential Geometry Fundamentals and Tools

continued

Special Co-distributions and their Properties

Annihilator of a distribution Δ ($\Delta^{\perp}$)

The set of all co-vectors which annihilates all vectors in $\Delta(x)$

$$\boxed{\Delta^{\perp}(x) = \left\{\omega^{*} \in \left(\mathbb{R}^{n}\right)^{*} \mid \left\langle \omega^{*}, v\right\rangle = 0 \ \forall v \in \Delta(x)\right\}}$$

is called the annihilator of the distribution Δ . **The annihilator of a distribution is a co-distribution.** The annihilator of a smooth distribution is not necessarily smooth.

Annihilator of a co-distribution Ω , ($\Omega^{\perp}$)

$$\boxed{\Omega^{\perp}(x) = \left\{v \in \mathbb{R}^{n} \mid \left\langle \omega^{*}, v\right\rangle = 0 \ \forall \omega^{*} \in \Omega(x)\right\}}$$

is the annihilator of a co-distribution Ω

The annihilator of a co-distribution is a distribution.

Differential Geometry Fundamentals and Tools

continued

Co-distribution invariant under the vector field f

The co-distribution Ω is invariant under the vector field f if and only if

$$\omega \in \Omega \Rightarrow L_f \omega \in \Omega \quad \textbf{where} \quad \boxed{\left(L_f \omega\right)\left(x\right) = \omega\left(x\right)\frac{\partial f}{\partial x}}$$

Sum of dimensions of a distribution and its annihilator

$$\dim\left(\Delta\right) + \dim\left(\Delta^{\perp}\right) = n$$

Inclusion properties

$$\Delta_1 \supset \Delta_2 \Leftrightarrow \Delta_1^{\perp} \subset \Delta_2^{\perp}$$

Annihilator of an intersection

$$\left(\Delta_1 \cap \Delta_2\right)^{\perp} \Leftrightarrow \Delta_1^{\perp} + \Delta_2^{\perp}$$

Differential Geometry Fundamentals and Tools

continued

Compatibility of the dimension of a distribution

If a distribution is spanned by the columns of a matrix F, the dimension of Δ at a point x_0 is equal to the rank of $F(x_0)$. If the entries of F are smooth functions of x then the annihilator of Δ is identified at each $x \in U$ by the set of row vectors ω^* satisfying the condition

$$\omega^* F(x) = 0$$

Compatibility of the dimension of a co-distribution

If a co-distribution Ω is spanned by the rows of a matrix W, whose entries are smooth functions of x, its annihilator is identified at each x by the set of vectors satisfying

$$W(x)v = 0 \Rightarrow \Omega^{\perp}(x) = \ker(W(x))$$

Differential Geometry Fundamentals and Tools

continued

Compatibility of the invariant property of annihilators

If a smooth distribution Δ is invariant under the vector field f then the co-distribution $\Omega = \Delta^{\perp}$ is also invariant under f

If a smooth co-distribution Ω is invariant under the vector field f then the distribution $\Delta = \Omega^{\perp}$ is also invariant under f

Condition of involutivity

A smooth distribution $\Delta(x) = span(f_1(x), \cdots, f_d(x))$ is involutive if and only if

$$\left[f_i, f_j\right] \in \Delta \quad \forall 1 \le i, j \le d$$

Differential Geometry Fundamentals and Tools

continued

Nonlinear Transformation of States

Transforming the coordinates in the state-space is also useful in the nonlinear case, in order to investigate dynamic properties of interest (e.g. reachability, observability, etc.) or to solve certain control problems. Of course, nonlinear coordinate transformations are usually applied to nonlinear state-space models.

Nonlinear coordinate transformation

A nonlinear change of coordinates is written as $z = \Phi(x)$ where Φ represents an $\mathbb{R}^n$ -valued function of n variables, i.e.

$$\Phi(x) = \begin{bmatrix} \Phi_1(x) \\ \vdots \\ \Phi_n(x) \end{bmatrix}$$

Φ is invertible, i.e. there exists a function Φ^{-1} such that

$$\Phi^{-1}(\Phi(x)) = x \ \forall x \in \mathbb{R}^n$$

Φ and Φ^{-1} are both smooth mappings, i.e. they have continuous partial derivatives of any order.

A transformation of this type is called a **global diffeomorphism** on $\mathbb{R}^n$

Differential Geometry Fundamentals and Tools

continued

Nonlinear Transformation of States

Sometimes, a transformation having both of these properties and being defined for all x is difficult to find. Thus, in most cases, one looks rather at transformations defined only in a neighborhood of a given point the existence of which is guaranteed by the inverse function theorem. **A transformation of this type is called a local diffeomorphism**. **In order to check whether a given transformation is a local diffeomorphism or not, the following result is very useful.**

Suppose Φ *is a smooth function defined on some subset* U *of* $\mathbb{R}^n$
Suppose the Jacobian matrix of Φ *is nonsingular at a point* $x = x_0$
then, on a suitable open subset U_0 *of* U *, containing* x_0 *,*
Φ *defines a local diffeomorphism.*

Modeling of NonLinear Faulty System

Given the NonLinear input affine system **(NLIAS)**

$$\begin{cases} \dot{x} & = & g_0(x) + \sum_{i=1}^{\ell_u} g_i(x)\, u_i + \sum_{i=1}^{\ell_f} \ell_i(x)\, f_i \\ y & = & h(x) \end{cases}$$

$f(x), g_i(x), l_i(x)$ are smooth vector fields

$h(x)$ is a nonlinear smooth mapping

$\ell_i(x)$ are defined as "**fault signatures**"

$x \in \mathrm{X}$	state vector,
$\mathrm{X} \in \mathbb{R}^n$	open subset
$y \in \mathbb{R}^p$	output vector
$u(t) \in \mathbb{R}^{\ell_u}$	input vector
$f(t) \in \mathbb{R}^{\ell_f}$	fault vector

Modeling of NonLinear Faulty System

Actuator fault model

$\ell_i(x) f_i$ can easily model a fault on the i^{th} actuator by assuming $\ell_i(x) = g_i(x)$

$x \in \mathrm{X}$	state vector,
$\mathrm{X} \in \mathbb{R}^n$	open subset
$y \in \mathbb{R}^p$	output vector
$u(t) \in \mathbb{R}^{\ell_u}$	input vector
$f(t) \in \mathbb{R}^{\ell_f}$	fault vector

In the following we will deal with fault on actuators (in particular loss of efficiency)

Remark: **with the faulty model previously presented it is possible also to model additive fault on sensors by mapping the additive sensors fault on the systems state.**

NonLinear Fundamental Problem of Residual Generation (NFPRG)

In the NFPRG a RG is sensitive to a given fault and non sensitive (decoupled) from the other remaining fault:

In this scheme a RG takes the observation $u(t), y(t)$ as inputs and generates a set of residual signals $r_i(t)$ with the following properties

- **when non fault is present, al the residuals $r_i(t), i = 1, \cdots, \ell_f$ decay asymptotically to zero**
- **the residual $r_i(t)$ are affected by the fault on the i^{th} component, all the other residuals $r_j(t), j \neq i$, are decoupled from this fault**

This coding set is also called a *dedicated residual set,* with this bank of RGs one can detect and isolate all fault simultaneously.

Another commonly used scheme is to make each RG sensitive to all but one fault: called *generalized residual set. By means of this set one cannot detect and isolate simultanus faults in two or more channels*

Geometric Approach to FDI of Nonlinear systems

Some tools from the NonLinear Geometric Approach are firstly introduced with reference to the nonlinear model (NLIAS) previously introduced

Definition 1: *the distribution* Δ *is said to be **conditioned invariant** (or $\left(h, f\right)$ invariant) for the NLIAS if it satisfies*

$$\left[g_i, \Delta \cap \ker\left\{dh\right\}\right] \subseteq \Delta \quad \textit{for all } i = 1, \cdots, \ell_u$$

where $g_0\left(x\right) = f\left(x\right)$, $\ker\left\{dh\right\}$ *is the distribution annihilating the differentials of the rows of the mapping* $h\left(x\right)$

For a given distribution P, the following algorithm is proposed in [Isidori, De Persis, 2001] to determine the smallest conditioned invariant distribution which contains P. *This distribution is denoted as* Σ_*^P

Geometric Approach to FDI of Nonlinear systems

Algorithm 1: *consider the following sequence of distributions*

$$\begin{cases} S_0 = \overline{P} \\ S_{k+1} = \overline{S}_k + \sum_{I=0}^{\ell_u} \left[g_i, \overline{S}_k \cap \ker\{dh\} \right] \end{cases}$$

where $\overline{S}$ *denotes the involutive closure of* S. *Suppose there is an integer* k^* *such that* $S_{k^*+1} = \overline{S}_{k^*}$. *Then* $\Sigma_*^P = \overline{S}_{k^*}$ *and* Σ_*^P *is involutive and is the smallest conditioned invariant containing* P. *Moreover any other distribution* Δ *which is involutive, contains* P *and is conditioned invariant satisfies* $\Delta \supset \Sigma_*^P$

According to [**De Persis, Isidori, 2001**], it is more convenient to work with a dual object, i.e. with a codistribution as defined next.

- **C. De Persis and A Isidori. A geometric approach to non–linear fault detection and isolation. *IEEE Transactions on Automatic Control, 45(6):853–865, June 2001.***

Geometric Approach to FDI of Nonlinear systems

Definition 2: *a codistribution* Ω *is said to be conditioned invariant if*

$$L_{g_i}\Omega \subset \Omega + span\left\{dh\right\}, i = 1, \cdots, \ell_u$$

where $span\left\{dh\right\}$ *is the codistribution spanned by the differentials of the rows of the mapping* $h\left(x\right)$.

REMARK

It has been shown [Isidori et al, 1981] that if $\Delta \cap \ker\left\{dh\right\}$ is a smooth distribution and $\Omega = \Delta^{\perp}$ is a smooth codistribution, then Ω satisfies definition 2.

Moreover it has been proved the following property:

if Σ_*^P satisfies algorithm 1, and nonsingular and $\Sigma_P^* \cap \ker\left\{dh\right\}$ is a smooth distribution, **it can be asserted that** $\left(\Sigma_*^P\right)^{\perp}$ **is the maximal conditioned invariant codistribution which is locally spanned by exact differentials and contained in** $P^{\perp}$

- **A. Isidori, A. J. Krener, C. Gori-Giorgi, and S. Monaco, "Nonlinear decoupling via feedback: A differential geometric approach,"** ***IEEE Trans. Automat. Contr., vol. AC-26, pp. 331–345, 1981.***

Geometric Approach to FDI of Nonlinear systems

Algorithm 2: *consider the NLIAS defined previously, and let* θ *denote a fixed codistribution. The observability codistribution algorithm (o.c.a.) that characterizes this non decreasing sequence of codistribution is given according to the following procedures*

$$\begin{cases} Q_0 = \theta \cap span\left\{dh\right\} \\ Q_{k+1} = \theta \cap \left(\sum_{I=0}^{\ell_u} \left[L_{g_i} Q_k + span\left\{dh\right\} \right] \right) \end{cases}$$

Suppose all codistributions of the sequences are nonsingular, so that there is an integer $k^* \leq n-1$ *such that* $Q_k = Q_{k^*}$ *for all* $k > k^*$ *and set* $\Omega^* = Q_k$.
The following notation is then used to stress the dependency of Ω^* *on* θ

$$\Omega^* = o.c.a.\left(\theta\right)$$

Definition 3: *The codistribution* Ω *is an* **observability codistribution** *for NLIAS if it is conditioned invariant (satisfies definition 2) and* $o.c.a.\left(\Omega\right) = \Omega$

Geometric Approach to FDI of Nonlinear systems

Remark: likewise, we may say that Δ is an *unobservability* ***distribution*** if its annihilator $\Omega = \Delta^{\perp}$ is an observability codistribution . *Then* $o.c.a.\left(\left(\Sigma_*^P\right)^{\perp}\right)$ *is by construction an observability codistribution contained in* $P^{\perp}$. As a matter of fact, it can be readily seen that $o.c.a.\left(\left(\Sigma_*^P\right)^{\perp}\right)$ is the largest codistribution having such properties

Proposition 1: *The codistribution* $o.c.a.\left(\theta\right)$ *is the maximal (in the sense of the codistribution inclusion)* ***observability codistribution*** *contained in* θ*. Suppose that the distribution* Σ_*^P *is well defined (i.e. satisfies algorithm 1). And nonsingular and that* $\Sigma_P^* \cap \ker\left\{dh\right\}$ *is a smooth distribution. Then* $o.c.a.\left(\left(\Sigma_*^P\right)^{\perp}\right)$ *is the maximal (in the sense of codistribution inclusion) observability codistribution which is locally spanned by exact differentials and contained in* $P^{\perp}$

- **deeper insight in : C. De Persis and A. Isidori, "On the observability codistributions of a nonlinear system,"** ***Syst. Control Lett., vol. 40, pp. 297–304, 2000.***

Geometric Approach to FDI of Nonlinear systems

Construction of a locally observable "quotient" system: next theorem states one of the important properties of the observability codistribution. We describe a change of coordinates, based on the properties of the observability codistribution algorithm, which is quite useful in addressing the problem of designing a residual generator.

Theorem 1: *Consider the NLIAS, previously introduced, with* $f_i = 0$ *, i.e.*

$$\underbrace{\begin{cases} \dot{x} = g_0(x) + \sum_{i=1}^{\ell_u} g_i(x)\, u_i \\ y = h(x) \end{cases}}_{\text{NLIAS with } f_i = 0}$$

Let Ω *be an observability codistribution. Let* n_1 *denote the dimension of* Ω*. Suppose that* Ω *Is locally spanned by exact differentials. Suppose that* $span\{dh\}$ *is non singular.*

it can be found a surjection Ψ_1 *and a function* Φ_1 *fulfilling*

$$\begin{aligned} \Omega^* \cap \text{span}\{dh\} &= \text{span}\{d(\Psi_1 \circ h)\} \\ \Omega^* &= \text{span}\{d(\Phi_1)\} \end{aligned}$$

Geometric Approach to FDI of Nonlinear systems

The functions $\Psi(y)$ *and* $\Phi(x)$ *are defined as*

$$\Psi(y) = \begin{pmatrix} \overline{y}_1 \\ \overline{y}_2 \end{pmatrix} = \begin{pmatrix} \Psi_1(y) \\ H_2 y \end{pmatrix}$$

$$\Phi(x) = \begin{pmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_3 \end{pmatrix} = \begin{pmatrix} \Phi_1(x) \\ H_2 h(x) \\ \Phi_3(x) \end{pmatrix}$$

where H_2 *is a selection matrix, i.e. a matrix in which any row has all 0 entries but one, which is equal to 1. Moreover* $\dim\left(\overline{x}_1\right) = \dim\left(\Phi_1(x)\right) = n_1$ *.*

In the new local coordinate , defined by the above functions (diffeomorphism) the NLIAS is described by the following equations

Geometric Approach to FDI of Nonlinear systems

In the new local coordinate the NLIAS has the form

$$\begin{cases} \dot{\overline{x}}_1 & = & n_1(\overline{x}_1, \overline{x}_2) + g_1(\overline{x}_1, \overline{x}_2)\, u \\ \dot{\overline{x}}_2 & = & n_2(\overline{x}_1, \overline{x}_2, \overline{x}_3) + g_2(\overline{x}_1, \overline{x}_2, \overline{x}_3)\, u \\ \dot{\overline{x}}_3 & = & n_3(\overline{x}_1, \overline{x}_2, \overline{x}_3) + g_3(\overline{x}_1, \overline{x}_2, \overline{x}_3)\, u \\ \overline{y}_1 & = & h(\overline{x}_1) \\ \overline{y}_2 & = & \overline{x}_2 \end{cases}$$

And in the new coordinates any vector field $p\left(x\right)$ in $\Omega^{\perp}$ can be expressed in the form

$$\begin{pmatrix} 0 & p_2^T\left(\overline{x}_1, \overline{x}_2, \overline{x}_3\right) & p_3^T\left(\overline{x}_1, \overline{x}_2, \overline{x}_3\right) \end{pmatrix}^T$$

Remark: the observability codistribution is a special type of conditioned invariant codistribution where the $\overline{x}_1$ -subsystem, in which $\overline{x}_2$ can be replaced by y_2 and viewed as an independent "input", namely the system

$$\begin{cases} \dot{\overline{x}}_1 & = & n_1\left(\overline{x}_1, \overline{y}_2\right) + g_1\left(\overline{x}_1, \overline{y}_2\right) u \\ \overline{y}_1 & = & h\left(\overline{x}_1\right) \end{cases}$$

satisfies the observability rank condition, **hence one can design an observer (filter) for the state $\overline{x}_1$**

Geometric Approach to FDI of Nonlinear systems

Theorem 2: ***the solvability of the NFPRG for the NLIAS***

$$\begin{cases} \dot{x} &= g_0(x) + \sum_{i=1}^{\ell_u} g_i(x)\, u_i + \sum_{i=1}^{\ell_f} \ell_i(x)\, f_i \\ y &= h(x) \end{cases}$$

Has a solution if and only if there exist observability codistribution $\Omega_i^* = o.c.a.\left(\left(\Sigma_*^{\mathcal{L}_i}\right)^{\perp}\right)$ *,*

with $\mathcal{L}_i = span\left\{l_1(x), \cdots, l_{i-1}(x), l_{i+1}(x), \cdots, l_{\ell_f}(x)\right\}$ *, such that* $\boxed{span\left\{\ell_i\right\} \subset \left(\Omega_i^*\right)^{\perp}}$

Fault f_i detectability and isolability condition

If such observability codistribution exist, then by using Theorem 1, the $\overline{x}_1$ ***- subsystem corresponding to each*** Ω_i^* ***is described in the new local coordinates by***

$$\begin{cases} \dot{\overline{x}}_1 &= n_1\left(\overline{x}_1, \overline{y}_2\right) + g_i\left(\overline{x}_1, \overline{y}_2\right) u_i + \ell_i\left(\overline{x}_1, \overline{y}_2\right) f_i \\ \overline{y}_1 &= h\left(\overline{x}_1\right) \end{cases}$$

Consequently it is now possible to design an observer, or a filter, which generates the estimate $\hat{\overline{x}}_1$ ***: the residual signal*** $r_i(t) = y_1 - h_1\left(\hat{\overline{x}}_1\right)$ ***is then sensitive to*** f_i ***and decoupled from the other faults***

Robust FDI with disturbance decoupling properties

Now it is straightforward taking into consideration disturbances (as wind …)

$$\begin{cases} \dot{x} &= n(x) + g(x)u + l(x)f + p(x)d \\ y &= h(x) \end{cases}$$

d **is the disturbance vector to be decoupled**.

Elements of d could be also the other fault to be decoupled.

If $P =$ distribution spanned by $p(x)$

1. **Determine the largest observability codistribution contained in** $P^{\perp}$ (i.e. Ω^*)
2. **If** $l(x) \not\subset (\Omega^*)^{\perp}$ **continue to next steps**, otherwise the fault is not detectable
3. **In a new (local) coordinate the model can be expressed as**: (*see next slide*)

Robust FDI with disturbance decoupling properties

In the new local coordinates the model has the form

$$\begin{cases} \dot{\overline{x}}_1 &= n_1(\overline{x}_1,\overline{x}_2) + g_1(\overline{x}_1,\overline{x}_2)u + \ell_1(\overline{x}_1,\overline{x}_2,\overline{x}_3)f \\ \dot{\overline{x}}_2 &= n_2(\overline{x}_1,\overline{x}_2,\overline{x}_3) + g_2(\overline{x}_1,\overline{x}_2,\overline{x}_3)u + \ell_2(\overline{x}_1,\overline{x}_2,\overline{x}_3)f + p_2(\overline{x}_1,\overline{x}_2,\overline{x}_3)d \\ \dot{\overline{x}}_3 &= n_3(\overline{x}_1,\overline{x}_2,\overline{x}_3) + g_3(\overline{x}_1,\overline{x}_2,\overline{x}_3)u + +\ell_3(\overline{x}_1,\overline{x}_2,\overline{x}_3)f + p_3(\overline{x}_1,\overline{x}_2,\overline{x}_3)d \\ \overline{y}_1 &= h(\overline{x}_1) \\ \overline{y}_2 &= \overline{x}_2 \end{cases}$$

$\overline{x}_1$ **- subsystem can be singled out**

$$\begin{cases} \dot{\overline{x}}_1 &= n_1(\overline{x}_1,\overline{y}_2) + g_1(\overline{x}_1,\overline{y}_2)u + \ell_1(\overline{x}_1,\overline{y}_2,\overline{x}_3)f \\ \overline{y}_1 &= h(\overline{x}_1) \end{cases}$$

System affected by the fault and decoupled from the disturbances. It can be exploited to design an observer and hence a RS. **In the following this subsystem will be exploited also for fault identification purposes by using adaptive filters**. **In this case the RS is defined as the difference between the fault and its estimate, i.e.** $f(t) - \hat{f}(t)$ **. Finally, when the diagnosis is based on a fault estimate it usual defined as FDD (Fault Detection and Diagnosis) instead of FDI**

The NonLinear Geometric Approach (NLGA) procedure

1. Determine Σ_*^P minimal conditioned invariant distribution containing : P

$$S_0 = \bar{P}; \quad S_{k+1} = \bar{S}_k + \sum_{i=0}^{\ell_u} [g_i, \bar{S}_k \bigcap Ker\{dh\}]$$

if $S_{k+1} = \bar{S}_k$ then $\boxed{\Sigma_*^P = \bar{S}_k}$

2. Determine Ω^* the largest observability codistribution contained in $P^\perp$:

$$Q_0 = \Theta \bigcap \; span\{dh\}; \quad Q_{k+1} = \Theta \bigcap \left(\sum_{i=0}^{\ell_u} L_{g_i} Q_k + \mathrm{span}\{\mathrm{dh}\} \right)$$

if $Q_{k+1} = Q_k$ then define $o.c.a.\left(\Theta\right) = Q_k$; $\boxed{\Omega^* = o.c.a.\left(\left(\Sigma_*^P\right)^\perp\right)}$

3. If $\boxed{\ell(x) \notin \left(\Omega^*\right)^\perp}$ fault f is detectable an a coordinate change can be exploited to determine the $\bar{x}_1$ - subsystem as previously described and recalled in the following

Remark: note that $(\Sigma_*^P)^\perp$ is the maximal conditioned invariant codistribution contained in $P^\perp$

The NonLinear Geometric Approach (NLGA) procedure

4. a surjection Ψ_1 and a function Φ_1 can be found

$$\Omega^* \cap \mathrm{span}\{dh\} = \mathrm{span}\{d(\Psi_1 \circ h)\}$$
$$\Omega^* = \mathrm{span}\{d(\Phi_1)\}$$

such that we have the local diffeomorphism

$$\Psi(y) = \begin{pmatrix} \bar{y}_1 \\ \bar{y}_2 \end{pmatrix} = \begin{pmatrix} \Psi_1(y) \\ H_2 y \end{pmatrix}; \quad \Phi(x) = \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{pmatrix} = \begin{pmatrix} \Phi_1(x) \\ H_2 h(x) \\ \Phi_3(x) \end{pmatrix}$$

allowing to compute

$$\begin{cases} \dot{\bar{x}}_1 = n_1(\bar{x}_1, \bar{x}_2) + g_1(\bar{x}_1, \bar{x}_2) u + \ell_1(\bar{x}_1, \bar{x}_2, \bar{x}_3) f \\ \dot{\bar{x}}_2 = n_2(\bar{x}_1, \bar{x}_2, \bar{x}_3) + g_2(\bar{x}_1, \bar{x}_2, \bar{x}_3) u + \ell_2(\bar{x}_1, \bar{x}_2, \bar{x}_3) f + p_2(\bar{x}_1, \bar{x}_2, \bar{x}_3) d \\ \dot{\bar{x}}_3 = n_3(\bar{x}_1, \bar{x}_2, \bar{x}_3) + g_3(\bar{x}_1, \bar{x}_2, \bar{x}_3) u + +\ell_3(\bar{x}_1, \bar{x}_2, \bar{x}_3) f + p_3(\bar{x}_1, \bar{x}_2, \bar{x}_3) d \\ \bar{y}_1 = h(\bar{x}_1) \\ \bar{y}_2 = \bar{x}_2 \end{cases}$$

(in blue the $\bar{x}_1$ - subsystem affected by the fault and decoupled from disturbances)

NLGA Actuator FDI for a general aviation aircraft

- **On board actuators FDI for a general aviation aircraft**

- **NLGA to FDI**

  - First complete NLGA FDI scheme with a 6DoF rigid body formulation
  - with tight coupled longitudinal and lateral–directional dynamics
  - Residual generators analytically decoupled from the vertical and lateral components of the wind (turbulence and gusts)

- **Tests by means of a Piper PA30 simulator**

  - Nonlinear 6DoF aircraft model, model of the measurement sensors, actuators and Dryden turbulence/wind-gust description
  - Aerodynamic coefficients by NASA

Aircraft Mathematical Description

The simulation model is made up of

- Aircraft 6DoF flight dynamics
- Engine model
- Model of I/O sensors
- Model of Servo Actuators
- Dryden Atmosphere Turbulence description
- Wind Gusts description
- Classical Autopilot

Standard Nomenclature

- $V, \alpha, Q, \theta, H, n$: TAS, Attack Angle, Pitch Rate, Elevation Angle, Altitude, rpm

- β, P, R, ϕ, ψ : Sideslip Angle, Roll Rate, Yaw Rate, Bank Angle, Heading Angle

- $\delta_e, \delta_a, \delta_r, \delta_{th}$: Elevator, Aileron, Rudder Deflection Angle and Throttle Aperture Percentage

- $F_x, F_y, F_z, M_x, M_y, M_z$: Total Force and Moment Components along body axes

6DoF Aircraft Dynamic

$$\dot{V} = F_x \frac{\cos\alpha\cos\beta}{m} + F_y \frac{\sin\beta}{m} + F_z \frac{\sin\alpha\cos\beta}{m}$$

$$\dot{\alpha} = \frac{-F_x \sin\alpha + F_z \cos\alpha}{mV\cos\beta} + Q - \left(P\cos\alpha + R\sin\alpha\right)\tan\beta$$

$$\dot{\beta} = \frac{-F_x \cos\alpha\sin\beta + F_y \cos\beta - F_z \sin\alpha\sin\beta}{mV} + P\sin\alpha + -R\cos\alpha$$

$$\dot{P} = \frac{M_x I_z + M_z I_{xz} + PQ\, I_{xz}\left(I_x - I_y + I_z\right)}{I_x I_z - I_{xz}^2} + \frac{QR\left(I_y I_z - I_{xz}^2 - I_z^2\right)}{I_x I_z - I_{xz}^2}$$

$$\dot{Q} = \frac{M_y + PR\left(I_z - I_x\right) - P^2 I_{xz} + R^2 I_{xz}}{I_y}$$

$$\dot{R} = \frac{M_x I_{xz} + M_z I_x + PQ\left(I_x^2 - I_x I_y + I_{xz}^2\right)}{I_x I_z - I_{xz}^2} + \frac{QR\, I_{xz}\left(-I_x + I_y - I_z\right)}{I_x I_z - I_{xz}^2}$$

$$\dot{\phi} = P + Q\sin\phi\tan\theta + R\cos\phi\tan\theta$$

$$\dot{\theta} = Q\cos\phi - R\sin\phi$$

$$\dot{\psi} = \frac{Q\sin\phi + R\cos\phi}{\cos\theta}$$

$$\dot{H} = V\cos\alpha\cos\beta\sin\theta - V\cos\theta\left(\sin\beta\sin\phi + \sin\alpha\cos\beta\cos\phi\right)$$

PA-30 engine model

$$\boxed{\dot{n} = \frac{BP(H)\cdot(1-\eta_{pr})}{I_{pr}n} - \frac{J_{v_1}}{I_{pr}}n - \frac{J_{v_2}}{I_{pr}}n^2 - \frac{J_{v_3}}{I_{pr}}n^3}$$

$\eta_{pr}, I_{pr} \Rightarrow$ propeller efficiency and inertiamoment

$J_{v_1}, J_{v_2}, J_{v_3} \Rightarrow$ polynomial coefficients of the engine shaft viscous friction

$$BP(H) = BP_0 \frac{\rho(H)}{\rho_0}\sqrt{\frac{T(H)}{T_0}} \text{ with } BP_0 = \delta_{th}\cdot P_c(n)$$

$BP(H), BP_0 \Rightarrow$ brake power of a single engine

$\rho(H), \rho_0 \Rightarrow$ air density

$T(H), T_0 \Rightarrow$ air temperature

$P_c(n) \Rightarrow$ engine power behavior with respect to n at full throttle

Overall thrust intensity $T_h = \dfrac{2\cdot BP(H)\cdot\eta_{pr}}{V\cos\alpha\cos\beta}$

Description of Turbulence and Wind Gusts

- ***Dryden Atmosphere Turbulence Model:*** *Dryden Turbulence* Model block of the Aerospace Blockset of MatlabR 7.0) ⇨ "light turbulence"

- ***Wind Gusts:*** w_u, w_v, w_w ⇨ independent air velocity components along aircraft body axes

  - According to the discrete wind gust formulation given in Military Specifications

  - "1 − cosine" shape with a *gust magnitude of 3 m/s and a gust length equal to the turbulence scale length in the* Dryden formulation (533.4 m)

Measurement Subsystems

- ***Command Surfaces Deflection Measurements:*** → **potentiometers**
  - **Errors**: bias and white noise
- ***Angular Rate Measurement (IMU): 3 gyroscopes of an Inertial Measurement Unit***
  - **Errors** : non unitary scale factor, Alignment error (random), g–sensitivity, Additive white noise, Gyro drift
- ***Attitude Angle Measurement: Digital filtering system based on a DSP processing*** both angular rate and accelerations provided by the **IMU**
  - **Errors** :
    - ✓A systematic error generated by the apparent vertical
    - ✓ A colored noise due to system structure and the environment influences
    - ✓Angular rate measurements provided by a gyroscope unit different from gyroscope device estimating angular rates (small drift vs larger bandwidths)

Measurement Subsystems and Servo Actuator Model

- ***Air Data System (ADS): ADS unit consists of an Air Data Computer (ADC)***
  - **Errors affecting the TAS**: Calibration error of differential pressure sensor, Additive colored noise induced by wind gusts and atmospheric turbulence, Additive white noise
  - **Errors affecting Altitude**: Calibration error of the static pressure sensor, Additive white noise
  - **Errors affecting attack and sideslip angle**: Calibration error affecting the wing boom sensors, Additive white noise

- ***Heading Reference System (HRS): magnetic compass cpl to a directional gyro***
  - **Errors:** a systematic error generated by a bias of the magnetic compass, white noise, additive colored noise

- ***Engine Shaft Rate Measurement****: incremental encoder* → *white noise.*

- ***Servo Actuator Models** : **second order linear models with saturations***

FDI model and residual generators design

Simulation Model not fulfilling NLGA assumption ⇨ approximations

- simplified aerodynamic: only main terms series expansion
- simplified engine model: linearization of the power with respect to the angular rate
- the second order coupling between the longitudinal and lateral–directional dynamics have been neglected
- the x –body axis component of the turbulence/wind-gusts has been neglected. (can be considered equivalent to a disturbance on the δ_{th} and its value is smaller than δ_{th} measurement noise)
- the rudder effect in the equation describing the β dynamics has been neglected (β dynamics will never be used).

Simplified Nonlinear Model for NLGA Residual Design

$$
\begin{aligned}
\dot{V} &= -\frac{\left(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha_2}\alpha^2\right)}{m}V^2 + n + g\left(\sin\alpha\cos\theta\cos\phi - \cos\alpha\sin\theta\right) + n + \frac{\cos\alpha}{m}\frac{t_p}{V}\left(t_0 + t_1 n_e\right)\delta_{th} + w_v \sin\alpha \\
\dot{\alpha} &= -\frac{\left(C_{L0} + C_{L\alpha}\alpha\right)}{m}V + n + \frac{g}{V}\left(\cos\alpha\cos\theta\cos\phi + \sin\alpha\sin\theta\right) + q_\omega + n + -\frac{\sin\alpha}{m}\frac{t_p}{V^2}\left(t_0 + t_1 n_e\right)\delta_{th} + \frac{\cos\alpha}{V}w_v \\
\dot{\beta} &= \frac{\left(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha_2}\alpha^2\right)\sin\beta + C_{Y\beta}\beta\cos\beta}{m}V + n + g\frac{\cos\theta\sin\phi}{V} + p_\omega\sin\alpha - r_\omega\cos\alpha + n + \\
&\quad -\frac{\cos\alpha\sin\beta}{m}\frac{t_p}{V^2}\left(t_0 + t_1 n_e\right)\delta_{th} + \frac{1}{V}w_\ell \\
\dot{p}_\omega &= \frac{\left(C_{l\beta}\beta + C_{lp}\,p_\omega\right)}{I_x}V^2 + \frac{\left(I_y - I_z\right)}{I_x}q_\omega r_\omega + \frac{C_{\delta_a}}{I_x}V^2\delta_a \\
\dot{q}_\omega &= \frac{\left(C_{m0} + C_{m\alpha}\alpha + C_{mq}\,q_\omega\right)}{I_y}V^2 + \frac{\left(I_z - I_x\right)}{I_y}p_\omega r_\omega + n + \frac{C_{\delta_e}}{I_y}V^2\delta_e + \frac{t_d}{I_y}\frac{t_p}{V}\left(t_0 + t_1 n_e\right)\delta_{th} \\
\dot{r}_\omega &= \frac{\left(C_{n\beta}\beta + C_{nr}\,r_\omega\right)}{I_z}V^2 + \frac{\left(I_x - I_y\right)}{I_z}p_\omega q_\omega + \frac{C_{\delta_r}}{I_z}V^2\delta_r \\
\dot{\phi} &= p_\omega + \left(q_\omega\sin\phi + r_\omega\cos\phi\right)\tan\theta \\
\dot{\theta} &= q_\omega\cos\phi - r_\omega\sin\phi \\
\dot{\psi} &= \frac{\left(q_\omega\sin\phi + r_\omega\cos\phi\right)}{\cos\theta} \\
\dot{n}_e &= t_n n_e^3 + \frac{t_f}{n_e}\left(t_0 + t_1 n_e\right)\delta_{th}
\end{aligned}
$$

$C_{(\bullet)} \Rightarrow$ aerodynamic coefficients; $t_{(\bullet)} \Rightarrow$ engine parameters

Elevator residual generator design (residual signal r_{δ_e})

Decoupling from wind and faults on aileron, rudder and throttle ➡

$$p(x) = \left[p_d(x) \quad g_2(x) \quad g_3(x) \quad g_4(x) \right] = \begin{bmatrix} \sin\alpha & 0 & 0 & 0 & \frac{\cos\alpha}{m}\frac{t_p}{V}(t_0 + t_1 n_e) \\ \frac{\cos\alpha}{V} & 0 & 0 & 0 & -\frac{\sin\alpha}{m}\frac{t_p}{V^2}(t_0 + t_1 n_e) \\ 0 & \frac{1}{V} & 0 & 0 & -\frac{\cos\alpha\sin\beta}{m}\frac{t_p}{V^2}(t_0 + t_1 n_e) \\ 0 & 0 & \frac{C_{\delta_a}}{I_x}V^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{t_d}{I_y}\frac{t_p}{V}(t_0 + t_1 n_e) \\ 0 & 0 & 0 & \frac{C_{\delta_r}}{I_z}V^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{t_f}{n_e}(t_0 + t_1 n_e) \end{bmatrix}$$

- $p_d(x)$ (first two columns) **is the wind disturbance distribution**
- $g_2(x), g_3(x), g_4(x)$ are the actuators fault distributions related to δ_a, δ_r

Hence, the closure of P is given by $\bar{P} = \left[P \quad e_{10} \right]$, where e_{10} is the 10^{th} column of the identity matrix

Elevator residual generator design (continued)

$$\ker\{dh\} = \varnothing \Rightarrow \Sigma_*^P = \bar{P} \Rightarrow \left(\Sigma_*^P\right)^\perp = \left(\bar{P}\right)^\perp$$

$$\left(\Sigma_*^P\right)^\perp = \left(\bar{P}\right)^\perp = \begin{bmatrix} \cos\alpha & -V\sin\alpha & 0 & 0 & -\dfrac{I_y}{mt_d} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

By observing that $\mathrm{span}\{dh\} = I_{10}$, it follows that $\Omega^* = \left(\Sigma_*^P\right)^\perp = \left(\bar{P}\right)^\perp$, hence $\left(\Omega^*\right)^\perp = \Sigma_*^P = \bar{P}$. **Since,** $\boxed{\ell(x) = g_1(x) \notin \left(\Omega^*\right)^\perp}$ **the fault is detectable**

The change of output coordinates gives

$$\Psi_1(y) = \left[\left(V\cos\alpha - \frac{I_y}{mt_d} q_\omega\right) \quad \phi \quad \theta \quad \psi\right]^{\mathrm{T}}$$

Note that only $\bar{x}_{11}$ is directly affected by the fault, in fact the other variables are not fed by the inputs.

Elevator residual generator design continued

In order to design the residual generator it is necessary to compute

$$\dot{\bar{x}}_{11} = \dot{V}\cos\alpha - V\sin\alpha\cdot\dot{\alpha} - \frac{I_y}{mt_d}\dot{q}_\omega =$$
$$-\frac{\left(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha_2}\alpha^2\right)}{m}V^2\cos\alpha - g\sin\theta + +V^2\sin\alpha\frac{\left(C_{L0} + C_{L\alpha}\alpha\right)}{m}n +$$
$$-Vq_\omega\sin\alpha - \frac{C_{\delta_e}}{mt_d}V^2\delta_e + -\frac{\left(C_{m0} + C_{m\alpha}\alpha + C_{mq}q_\omega\right)}{mt_d}V^2 - \frac{\left(I_z - I_x\right)}{mt_d}p_\omega r_\omega$$

from which the elevator residual generator is given by (measurement filter form)

$$\begin{cases} \dot{\xi}_1 = \left[-(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha^2}\alpha^2)\cos\alpha + (C_{L0} + C_{L\alpha}\alpha)\sin\alpha\right]\dfrac{V^2}{m} \\ +g\sin\theta - qV\sin\alpha - \left[(C_{m0} + C_{m\alpha}\alpha) + C_{mq}q\right]\dfrac{V^2}{md_t} - \dfrac{C_{\delta_e}}{md_t}V^2\delta_e \\ +K_{\delta_e}\left(V\cos\alpha - \dfrac{I_y}{md_t}q - \xi_1\right) \\ r_{\delta_e} = \left(V\cos\alpha - \dfrac{I_y}{md_t}q - \xi_1\right) \quad \text{with } K_{\delta_e} > 0 \end{cases}$$

Aileron and Rudder residual generator

Determination of $\bar{P}$, and subsequent computations similar to r_{δ_e} case

$\ell(x) \notin \left(\Omega^*\right)^{\perp}$ fulfilled $\Rightarrow$ with $K_{\delta_a} > 0, K_{\delta_r} > 0$

The aileron residual generator $r_{\delta_a}(t)$ is given by

$$\begin{cases} \dot{\xi}_2 = \dfrac{\left(C_{l\beta}\beta + C_{lp}p_\omega\right)}{I_x}V^2 + \dfrac{\left(I_y - I_z\right)}{I_x}q_\omega r_\omega + \dfrac{C_{\delta_a}}{I_x}V^2\delta_a + K_{\delta_a}\left(p_\omega - \xi_2\right) \\ r_{\delta_a} = p_\omega - \xi_2 \end{cases}$$

The rudder residual generator $r_{\delta_r}(t)$ is given by

$$\begin{cases} \dot{\xi}_3 = \dfrac{\left(C_{n\beta}\beta + C_{nr}r_\omega\right)}{I_z}V^2 + \dfrac{\left(I_x - I_y\right)}{I_z}p_\omega q_\omega + \dfrac{C_{\delta_r}}{I_z}V^2\delta_r + K_{\delta_r}\left(r_\omega - \xi_3\right) \\ r_{\delta_r} = r_\omega - \xi_3 \end{cases}$$

Throttle residual generator design (residual signal $r_{\delta_{th}}(t)$)

Determination of $\bar{P}$, and subsequent computations similar to r_{δ_e} case

$\ell(x) \notin \left(\Omega^*\right)^{\perp}$ fulfilled $\Rightarrow$ with $K_{\delta_a} > 0, K_{\delta_r} > 0$

Moreover it can be shown that $\Phi_1(x) = \left[\left(V\cos\alpha\right) \quad \phi \quad \theta \quad \psi \quad n_e\right]^{\mathrm{T}}$

Note that both $\bar{x}_{11}$ and $\bar{x}_{15}$ are affected by the fault, leading to two possible throttle residual generators

$$\boxed{\begin{cases} \dot{\xi}_4 = \dot{\bar{x}}_{15} = t_n n_e^3 + \dfrac{t_f}{n_e}\left(t_0 + t_1 n_e\right)\delta_{th} + K_{\delta_{th}}\left(n_e - \xi_4\right) \\ r_{\delta_{th}} = n_e - \xi_4 \qquad\qquad K_{\delta_{th}} > 0 \end{cases}}$$

$$\begin{cases} \dot{\xi}_4' = \dot{\bar{x}}_{11} = -\dfrac{\left(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha_2}\alpha^2\right)}{m}V^2\cos\alpha - g\sin\theta + \\ +V^2\sin\alpha\dfrac{\left(C_{L0} + C_{L\alpha}\alpha\right)}{m} - Vq_\omega\sin\alpha + \dfrac{t_p}{mV}\left(t_0 + t_1 n_e\right)\delta_{th} + K_{\delta_{th}}'\left(V\cos\alpha - \xi_4'\right) \\ r_{\delta_{th}}' = V\cos\alpha - \xi_4' \qquad\qquad K_{\delta_{th}}' > 0 \end{cases}$$

$r_{\delta_{th}}$ is characterized by a fewer number of parameters with respect to $r_{\delta_{th}}'$

Hence $r_{\delta_{th}}$ is preferable to cope with robustness requirements

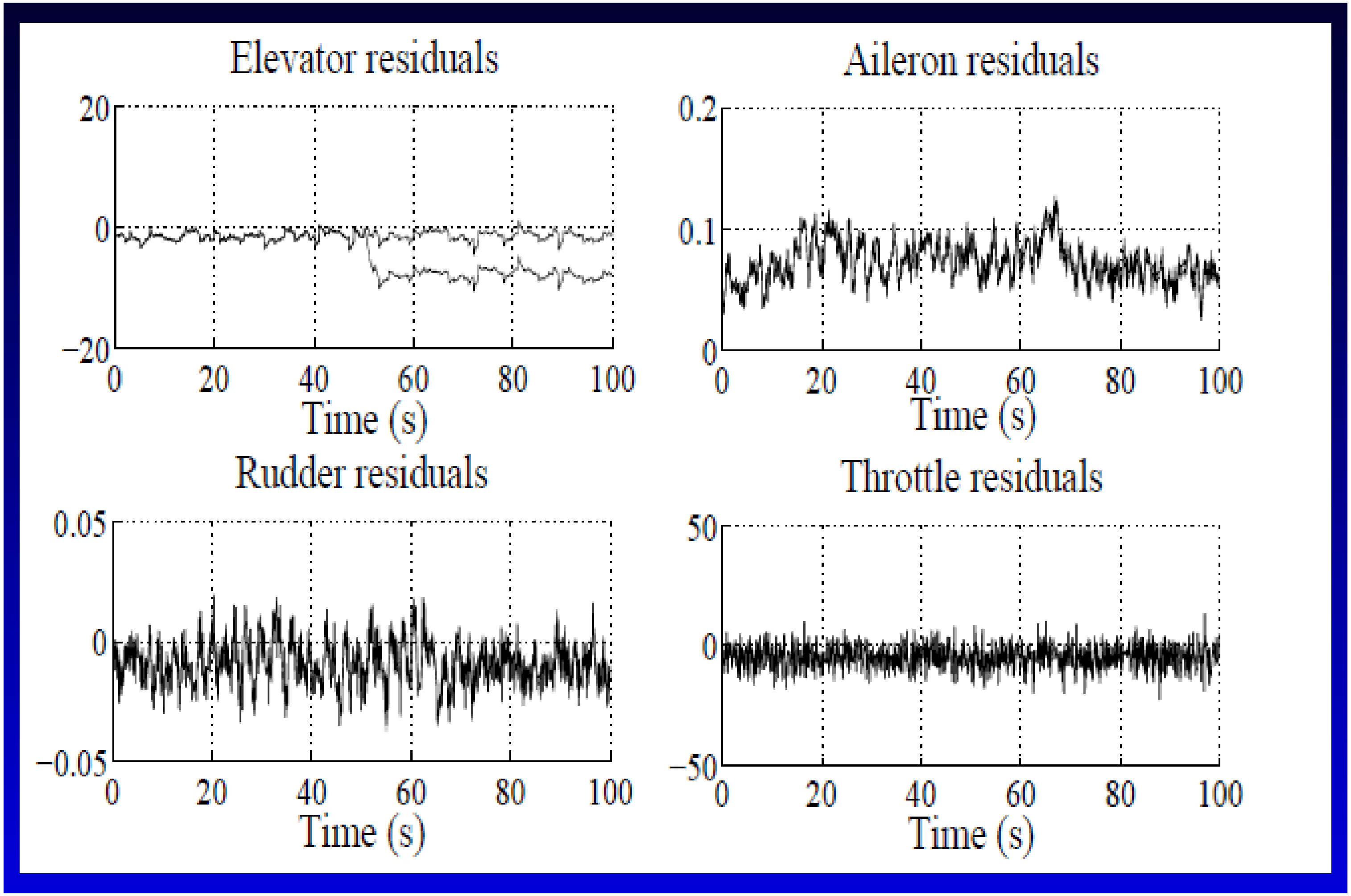
Results

Flight condition ⇒ **coordinate turn**

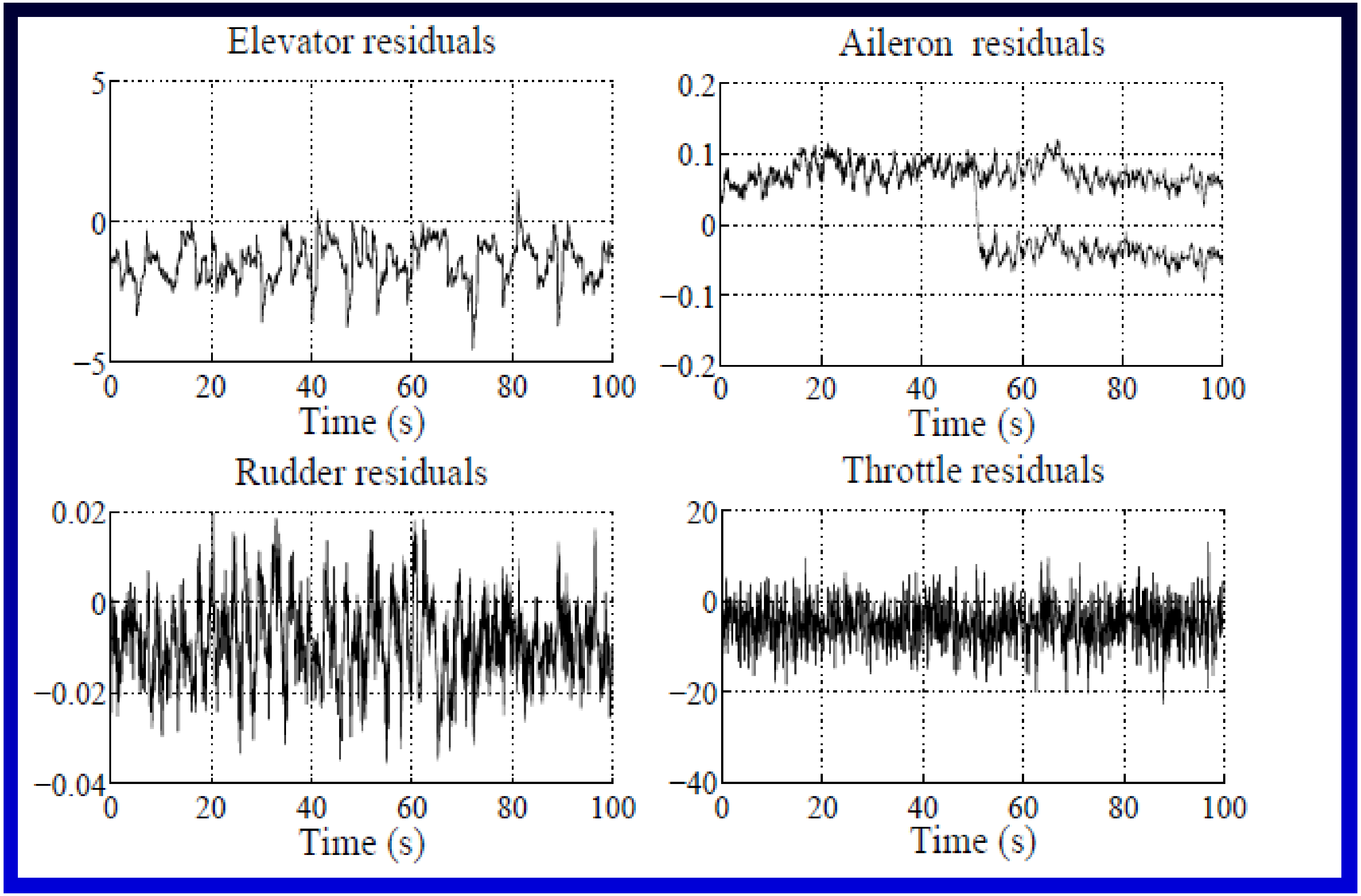
- radius of curvature $1000m, V = 50\dfrac{m}{s}, H = 330m, \gamma = 0^{\circ}, flap = 0^{o}$

- **FDI technique**: bank of 4 residual generators
- **Residual generator** decoupled from vertical and lateral wind gusts and three of the 4 input signals (hence insensitive to the corresponding fault signals)
- **Single faults in the actuator: positive and negative step function** (10 for command surfaces and 5% for throttle)
- **thresholds values, fixed in fault–free conditions**, depend on the residual error amount due to measurement errors, model approximations and disturbance signals that are not completely de–coupled
  - Margin of 10% between the positive and negative thresholds and the maximum and minimum values of the fault–free residual signals were respectively imposed
- Gains equal to one (good robustness)

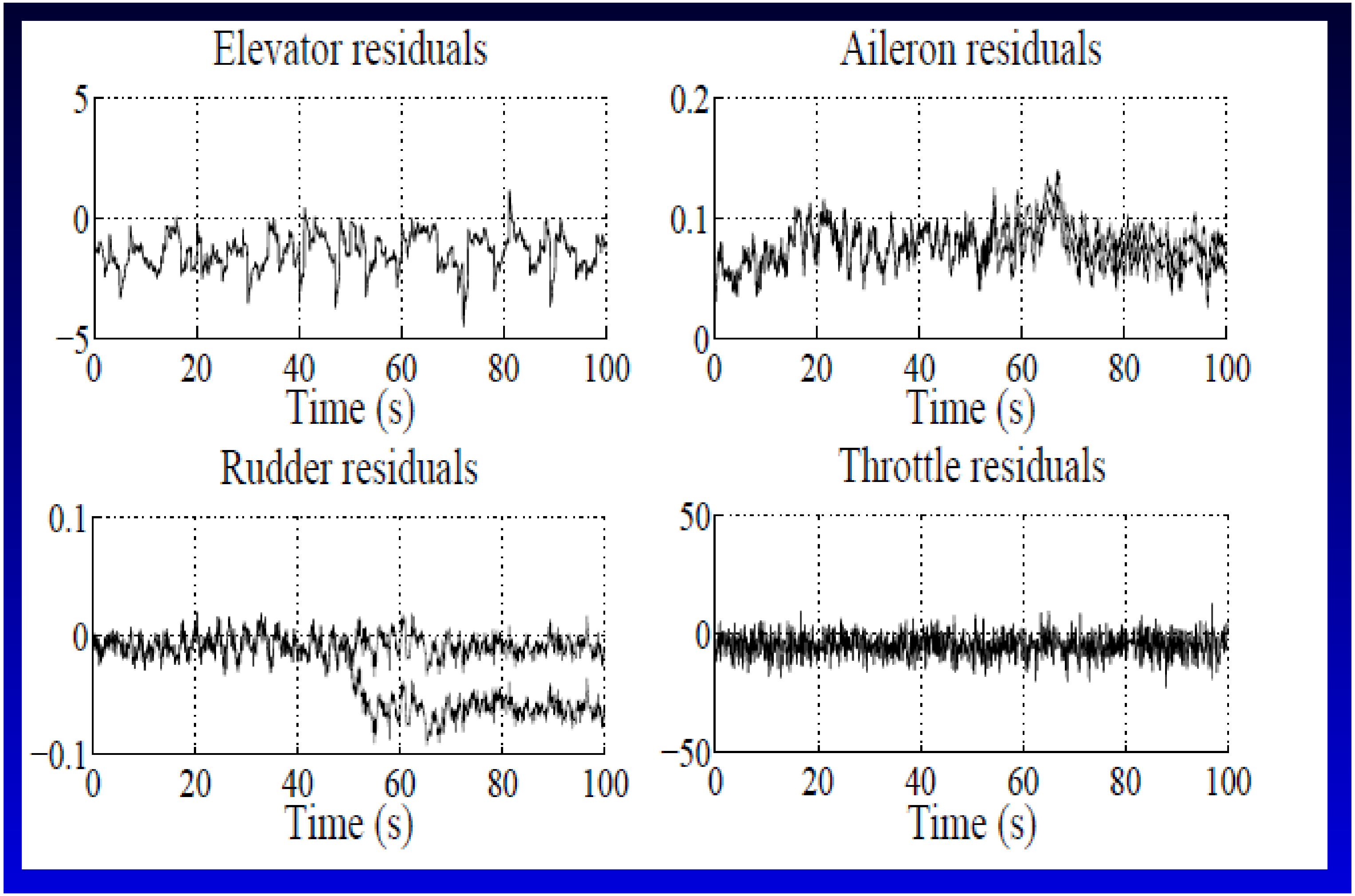
Fault on elevator: $f_{\delta_e} = 0.02$ rad



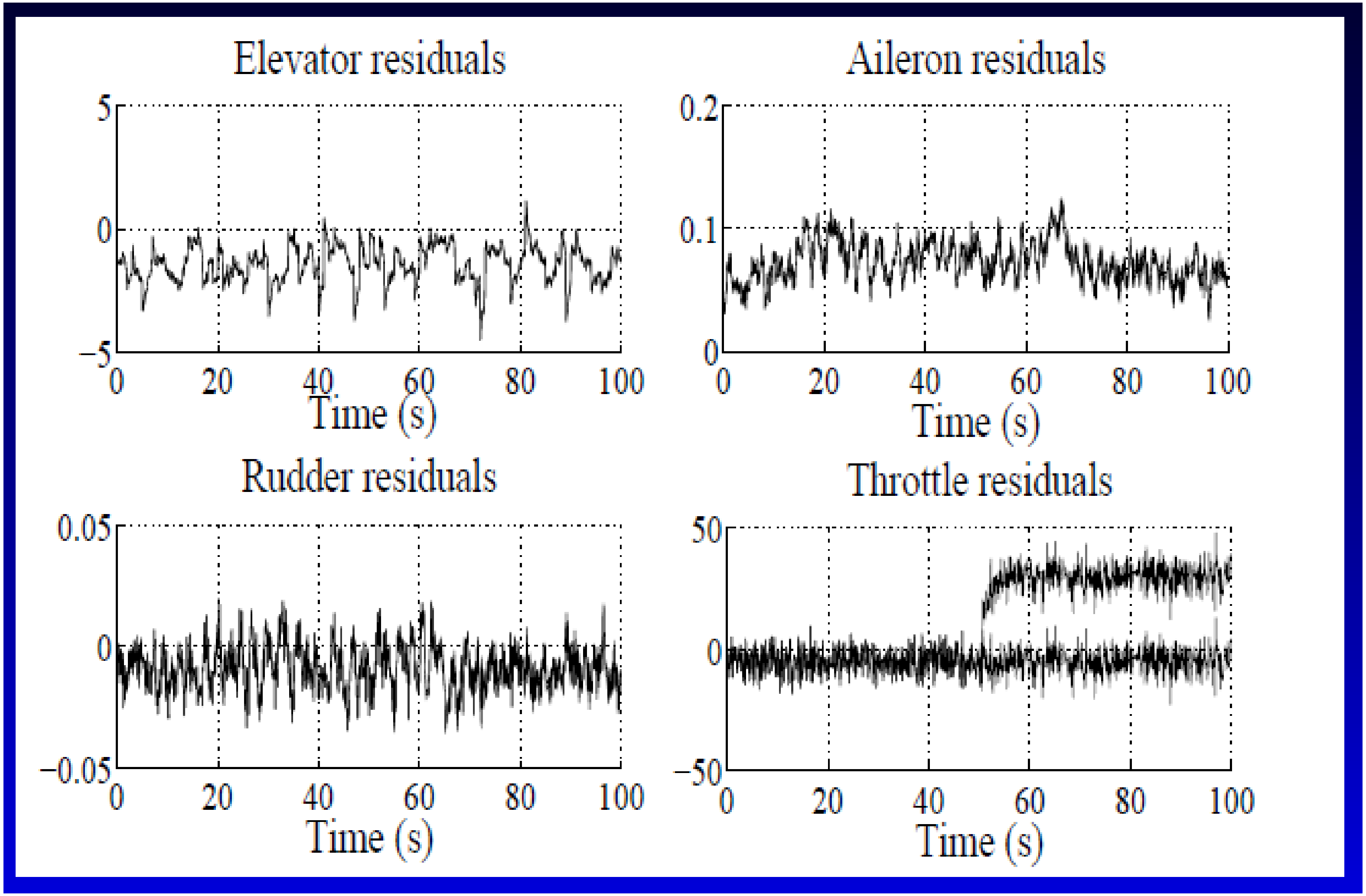
Fault on aileron: $f_{\delta_a} = 0.02$ rad



Fault on rudder: $f_{\delta_a} = 0.02$ rad



Fault on throttle: $f_{\delta_{th}} = 5\%$ rad



Motivations for implementing flight FTCS

- **Conventional feedback control design for a complex system: unsatisfactory performance, or even instability, in the event of malfunctions in actuators**

  (or other system components)

  - **new approaches have been developed in order to tolerate component malfunctions, while maintaining desirable stability and performance properties: FTCS (Fault Tolerant Control System)**

- **A significant amount of research on FTCS was motivated by aircraft flight control**. **In such systems, the consequences of a minor fault could be catastrophic as in the case of Delta Flight 1080 (April 12, 1977) (McMahan, 1978; Montoya, 1983), where the elevator got a fault and the pilot had been given no indication on this malfunction.**

- **A system for aiding pilots by providing automatic fault accommodation is therefore highly desirable for both civil and military aircrafts**.

- In the following the case of aircraft actuators' fault will be concerned

AFTCS by means of FDD

- An AFTCS relies heavily on real time schemes, such FDI or FDD, which provide the most up-to-date information about the true status of the system often using dynamic observers or filters

- FDI schemes using the residual generators have been deeply investigated and can be used for a logic-based switching controller

- FDD schemes are a more challenging topic because they provide fault detection, isolation and fault estimation. In an AFTCS, fault estimates are used for further feedback loops.

Active Fault Tolerant Control: FDD *module*

FDD procedure and module relies on Adaptive Filters designed for the sub-systems given by the NonLinear Geometric Approach coordinate change .
We will denote these Adaptive Filters as NLGA-AF

- **This filters are disturbance decoupled** thanks to **NLGA** which **allows to determine** equivalent representation of the aircraft model highlighting a **subsystem affected by the fault and decoupled by the disturbances and the other faults**

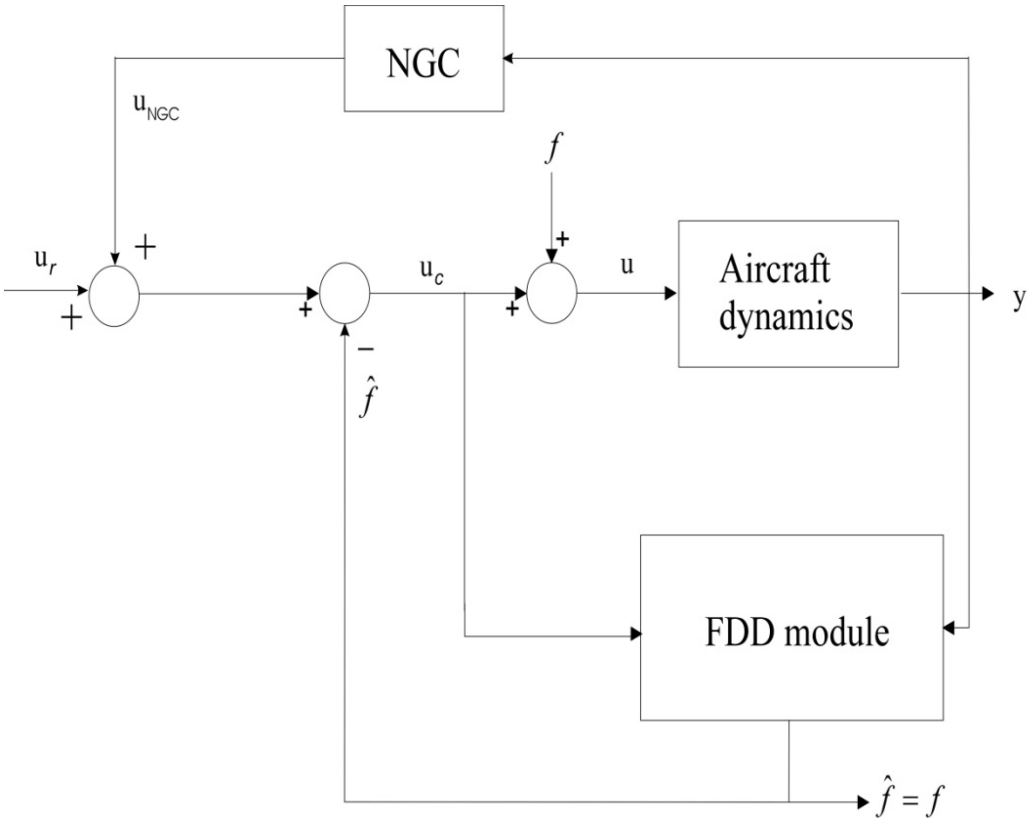
- **Adaptive filter structure** based on the **least-squares algorithm with forgetting factor are designed to obtain a fault estimate**

- **For the usual case of step function of time, the convergence of the fault estimate to the actual fault size has been proved in**

[P. Castaldi et al.,"Design of residual generators and adaptive filters for the FDI of aircraft model sensors". Control Engineering Practice.Vol. 18, pp. 449 - 459. May 2010]

- **Summarizing: the proposed AFTCS is based on the feedback of the estimated fault signal, which is obtained by the adaptive filters decoupled from disturbances (wind) thanks to NLGA**

AFTCS using the FDD module



Logic diagram of the integrated AFTCS strategy

- Novel FDD which estimates correctly the actual fault
- **After this correction, the current NGC module provides the exact tracking of the reference signal**
- **One of the main advantages of this strategy is that a structure of logic-based switching controller is not required**
- As usual in FDD, fault conditions are assumed to not modify the system structure, thus guaranteeing the global stability

FDD synthesis model & NLGA-AF design

We use the same approximation exploited in the design of the residual generators for FDI module

- **simplified aerodynamics** (main terms series expansion)
- simplified engine model (linearizing power with respect to the angular rate)
- longitudinal and lateral dynamics 2nd order coupling neglected
- x–body axis turbulence/wind-gusts component neglected
- rudder effect on β dynamics neglected

Refs:

M. Bonfè, P. Castaldi, W. Geri, and S. Simani, "Fault Detection and Isolation for On–Board Sensors of a General Aviation Aircraft," ***International Journal of Adaptive Control and Signal Processing, vol. 20, pp. 381–408, October 2006.***

M. Benini, M. Bonfè, P. Castaldi, W. Geri, and S. Simani, "Design and Analysis of Robust Fault Diagnosis Schemes for a Simulated Aircraft Model," ***Journal of Control Science and Engineering, vol. 2008, pp. 1–18, 2008.***

M. Bonfè, P. Castaldi, W. Geri, and S. Simani, "Nonlinear Actuator Fault Detection and Isolation for a General Aviation Aircraft," ***Space Technology – Space Engineering, Telecommunication, Systems Engineering and Control, vol. 27, pp. 107–113, December***

FDD Assumption

- **The new proposed FDD scheme can be applied only if the fault detectability condition holds and the following new constraints are satisfied**:
- the $\bar{x}_1$ –subsystem is independent from the $\bar{x}_3$ state components
- **the fault is a step function of the time** (loss of efficiency)
- **there exists a proper scalar component of the state vector such that the corresponding scalar component of the output vector is and the following relation holds**

$$\dot{\bar{y}}_{1s}(t) = M_1(t) \cdot f + M_2(t); \quad M_1(t) \neq 0, \forall t \geq 0$$

- **The structure of** $M_1(t)$ **and** $M_2(t)$ **is obtained by means of NLGA** . **It is straightforward to express the residual dynamics previously computed in this form, highlighting the fault** f . Moreover they can be computed for each time instant

- **The assumption are satisfied , see :**

  [P. Castaldi et al.,“ Design of residual generators and adaptive filters for the FDI of aircraft model sensors”. Control Engineering Practice.Vol. 18, pp. 449 - 459. May 2010]

FDD problem and solution

FDD Problem: ***The design of an adaptive filter is required, with reference to the system model , in order to perform an estimation , which asymptotically converges to the magnitude of the fault .***

- **Solution: the proposed adaptive filter (least-squares)**

$$\begin{cases} \dot{\breve{M}}_1 = -\lambda \breve{M}_1 + M_1, & \breve{M}_1(0) = 0 \\ \dot{\breve{M}}_2 = -\lambda \breve{M}_2 + M_2, & \breve{M}_2(0) = 0 \\ \dot{\breve{\bar{y}}}_{1s} = -\lambda \breve{\bar{y}}_{1s} + \bar{y}_{1s}, & \breve{\bar{y}}_{1s}(0) = 0 \end{cases}$$

Data low-pass filtering

$$\begin{cases} \hat{\bar{y}}_{1s} = \breve{M}_1 \hat{f} + \breve{M}_2 + \lambda \breve{\bar{y}}_{1s} \\ \epsilon = \dfrac{1}{N^2}\left(\bar{y}_{1s} - \hat{\bar{y}}_{1s}\right) \end{cases}$$

output estimation

normalized estimation error

$$\begin{cases} \dot{P} = \beta P - \dfrac{1}{N^2} P^2 \breve{M}_1^2, & P\left(0\right) = P_0 > 0 \\ \dot{\hat{f}} = P \epsilon \breve{M}_1, & \hat{f}\left(0\right) = 0 \end{cases}$$

adaptation law

$\lambda > 0; \beta \geq 0; N^2 = 1 + \breve{M}_1^2$ **Bandwidth; forgetting factor ; normalization factor**

NLGA-AF design: fault on elevator

Design of the NLGA-AF, the dynamic of the sub-system affected by elevator fault is

$$\dot{\bar{x}}_{11} = \dot{V}\cos\alpha - V\sin\alpha\cdot\dot{\alpha} - \frac{I_y}{mt_d}\dot{q}_\omega =$$

$$-\frac{\left(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha_2}\alpha^2\right)}{m}V^2\cos\alpha - g\sin\theta + +V^2\sin\alpha\frac{\left(C_{L0} + C_{L\alpha}\alpha\right)}{m} +$$

$$-Vq_\omega\sin\alpha - \frac{C_{\delta_e}}{mt_d}V^2\left(\delta_e + f_{\delta_e}\right) + -\frac{\left(C_{m0} + C_{m\alpha}\alpha + C_{mq}q_\omega\right)}{mt_d}V^2 - \frac{\left(I_z - I_x\right)}{mt_d}p_\omega r_\omega$$

from which the elevator NLGA_AF is given by

$$\begin{cases} \dot{\bar{y}}_{1s,e} = M_{1e}\cdot f_{\delta_e} + M_{2e} \\ M_{1e} = -\dfrac{C_{\delta_e}}{mt_d}V^2 \\ M_{2e} = \dfrac{V^2}{m}\left(-\left(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha_2}\alpha^2\right)\cos\alpha + \left(C_{L0} + C_{L\alpha}\alpha\right)\sin\alpha - \dfrac{\left(C_{m0} + C_{m\alpha}\alpha + C_{mq}q_\omega\right)}{t_d}\right) \\ \qquad -g\sin\theta - V\sin\alpha\, q_\omega - \dfrac{\left(I_z - I_x\right)}{mt_d}p_\omega r_\omega - \dfrac{C_{\delta_e}}{mt_d}V^2\,\delta_e \end{cases}$$

NLGA-AF estimation of aircraft faults

Fault on Elevator

$$\bar{x}_{1s} = \bar{x}_{11} = \left(V\cos\alpha - \frac{I_y}{mt_d} q_\omega \right) \Rightarrow \begin{cases} \dot{\bar{y}}_{1s,e} = M_{1e} \cdot f_{\delta_e} + M_{2e} \\ M_{1e} = -\frac{C_{\delta_e}}{mt_d} V^2 \\ M_{2e} = \frac{V^2}{m}\Bigg(-\left(C_{D0} + C_{D\alpha}\alpha + C_{D\alpha_2}\alpha^2 \right)\cos\alpha + \\ \quad + \left(C_{L0} + C_{L\alpha}\alpha \right)\sin\alpha \\ \quad - \frac{\left(C_{m0} + C_{m\alpha}\alpha + C_{mq} q_\omega \right)}{t_d} \Bigg) \\ \quad - g\sin\theta - V\sin\alpha\, q_\omega - \frac{\left(I_z - I_x \right)}{mt_d} p_\omega r_\omega - \frac{C_{\delta_e}}{mt_d} V^2 \delta_e \end{cases}$$

Fault on Aileron

$$\bar{x}_{1s} = \bar{x}_{11} = p_\omega \quad \Rightarrow \quad \begin{cases} \dot{\bar{y}}_{1s,a} = M_{1a} \cdot f_{\delta_a} + M_{2a} \\ M_{1a} = \frac{C_{\delta_a}}{I_x} V^2 \\ M_{2a} = \frac{\left(C_{l\beta}\beta + C_{lp} p_\omega \right)}{I_x} V^2 + \frac{\left(I_y - I_z \right)}{I_x} q_\omega r_\omega + \\ \quad + \frac{C_{\delta_a}}{I_x} V^2 \delta_a \end{cases}$$

NLGA-AF estimation of aircraft faults

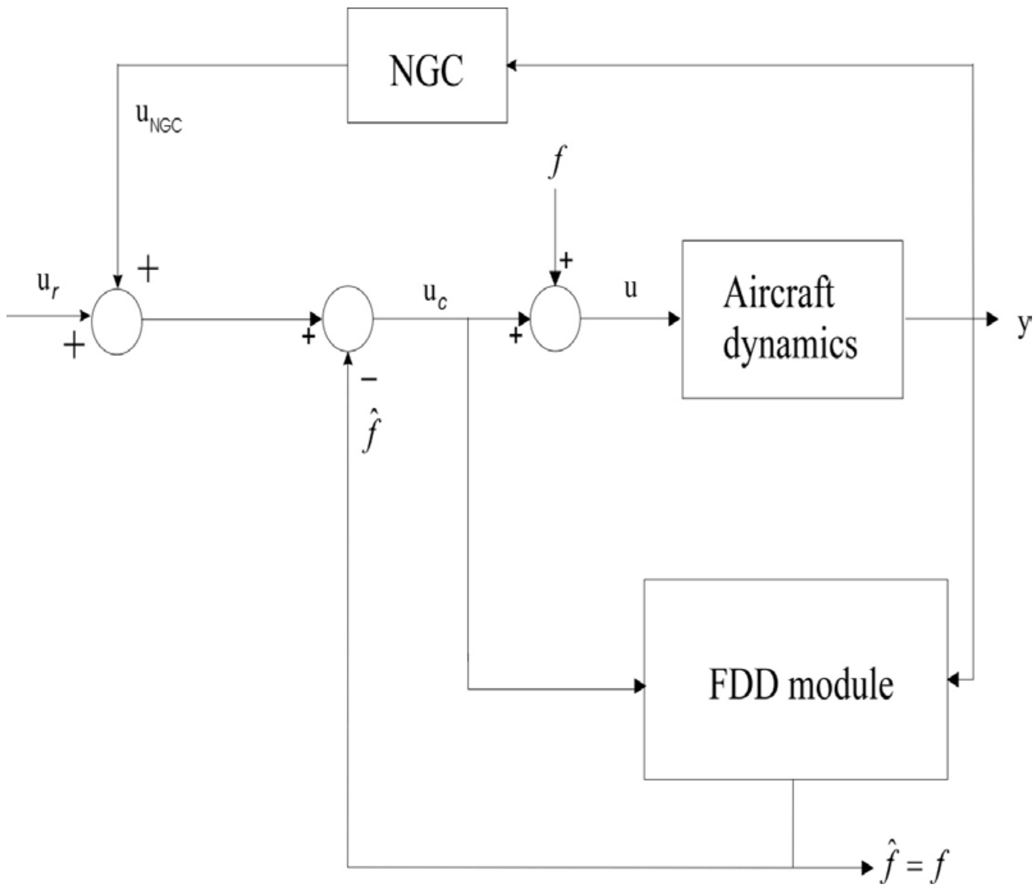
Fault on rudder

$$\bar{x}_{1s} = \bar{x}_{11} = r_\omega \quad \Rightarrow \quad \begin{cases} \dot{\bar{y}}_{1s,r} = M_{1r} \cdot f_{\delta_r} + M_{2r} \\ M_{1r} = \dfrac{C_{\delta_r}}{I_z} V^2 \\ M_{2r} = \dfrac{\left(C_{n\beta}\beta + C_{nr} r_\omega\right)}{I_z} V^2 + \dfrac{\left(I_x - I_y\right)}{I_z} p_\omega q_\omega + \\ \qquad + \dfrac{C_{\delta_r}}{I_z} V^2 \delta_r \end{cases}$$

Fault on Throttle

$$\bar{x}_{1s} = \bar{x}_{15} = n_e \quad \Rightarrow \quad \begin{cases} \dot{\bar{y}}_{1s,th} = M_{1th} \cdot f_{\delta_{th}} + M_{2th} \\ M_{1th} = \dfrac{t_f}{n_e}\left(t_0 + t_1 n_e\right) \\ M_{2th} = t_n n_e^3 + \dfrac{t_f}{n_e}\left(t_0 + t_1 n_e\right)\delta_{th} \end{cases}$$

AFTCS scheme



u_r : reference input (*e.g.* the reference trajectory)

u : actuated input

u_c : controlled input

u_{NGC}: feedback signal from the NGC system

NGC: Navigation, Guidance and Control system;

y : controlled output (e.g. the aircraft trajectory)

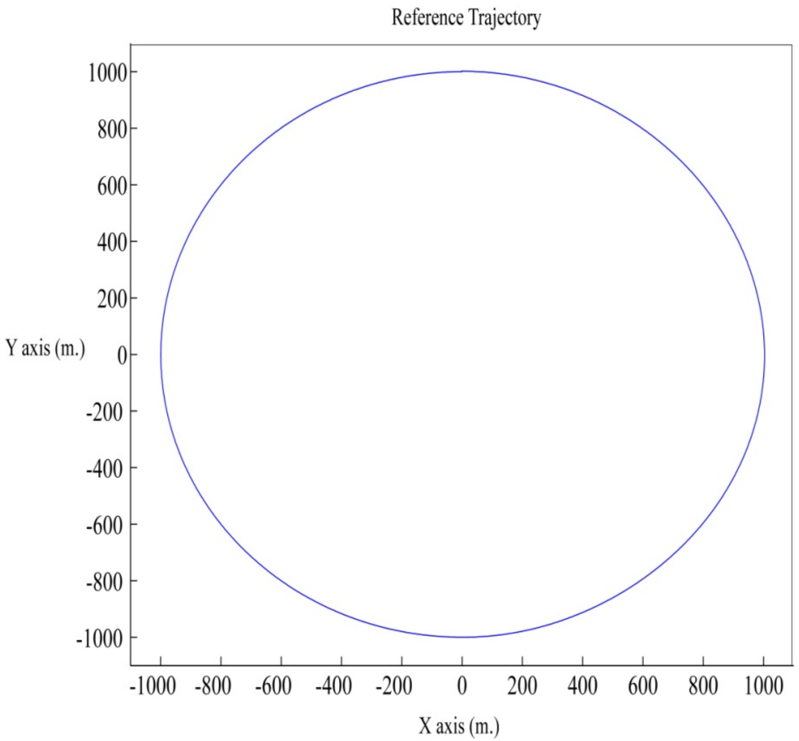
f : actuator fault

$\hat{f}$: **estimated actuator fault.**

Case Study

Coordinated turn (circular trajectory)

- **tight coupled longitudinal and lateral-directional dynamics**
- radius of curvature of 1000 m
- **true air speed of 52.36 m/s (120 s to cover the whole circular trajectory)**
- altitude of 330 m



Aileron actuator fault

- with a size of -5^0
- **starting at time** $t = 60s$ (i.e. **in the middle of the circular trajectory**)

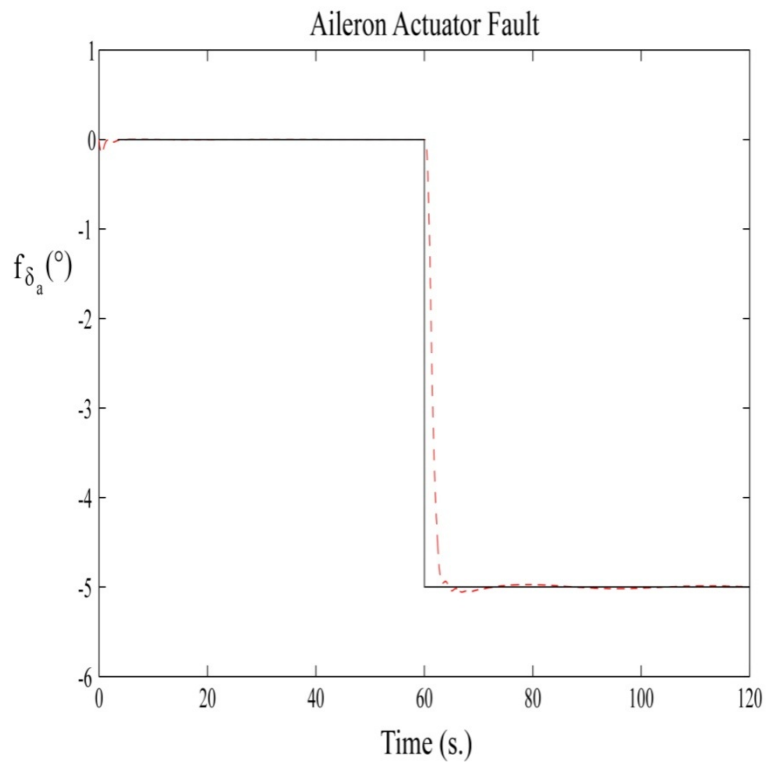
Case Study: no wind condition, fault on aileron $f_{\delta_a} = -5^\circ$

- **Aileron Fault Estimate**

(in case of parameter uncertainties: bias eliminating thanks to fault feedback)

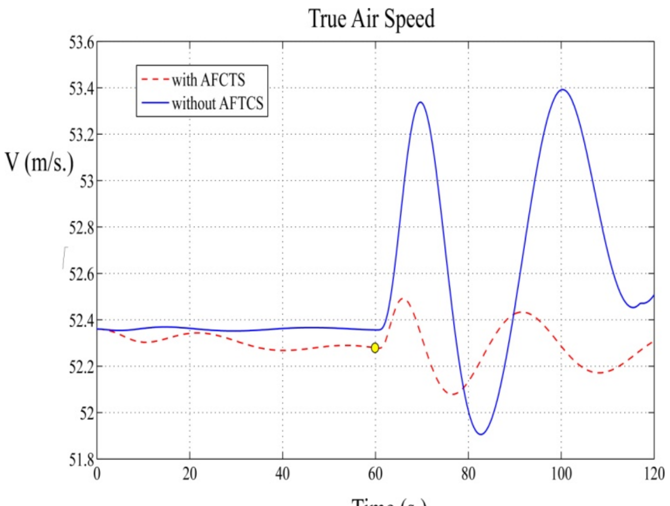
Black line: f_{δ_a} actual fault

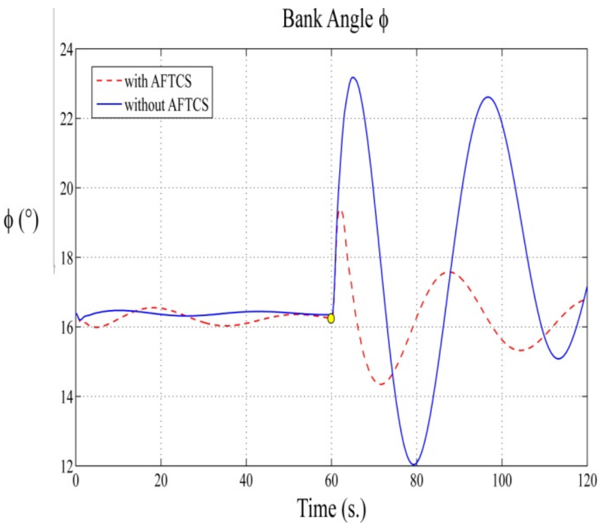
Red line : $\hat{f}$ **fault estimate**

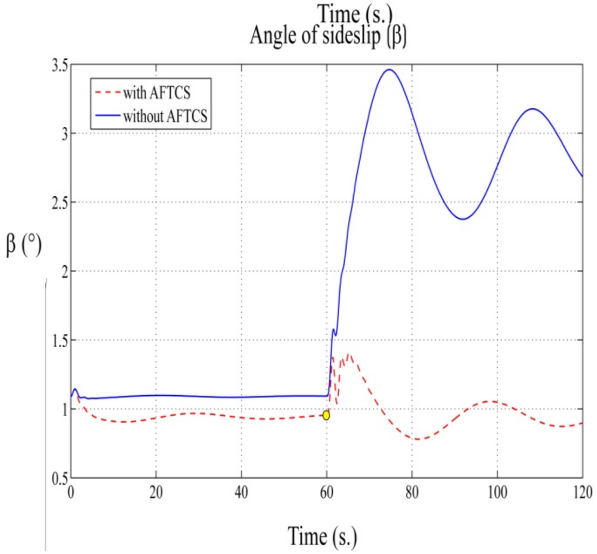


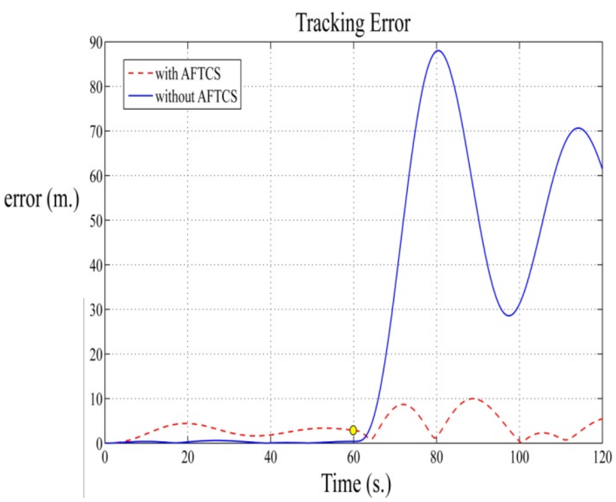
Case study : no wind condition, fault on aileron $f_{\delta_e} = -5^{\circ}$

red :with AFTCS blue: without AFTCS



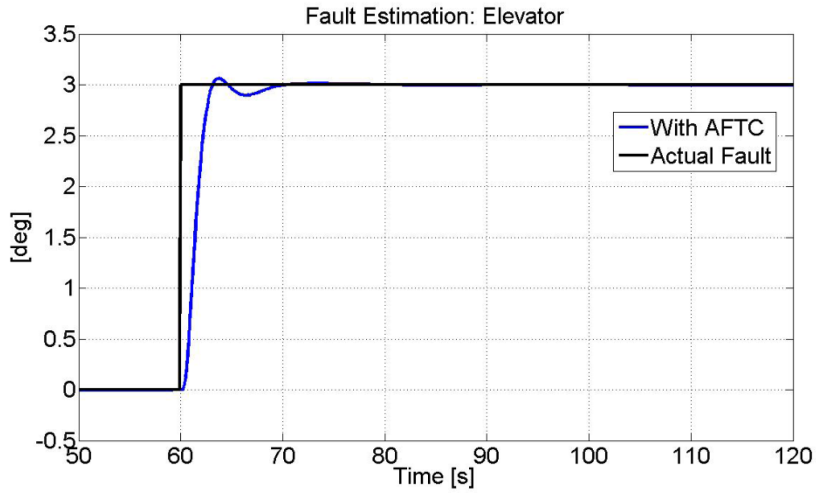


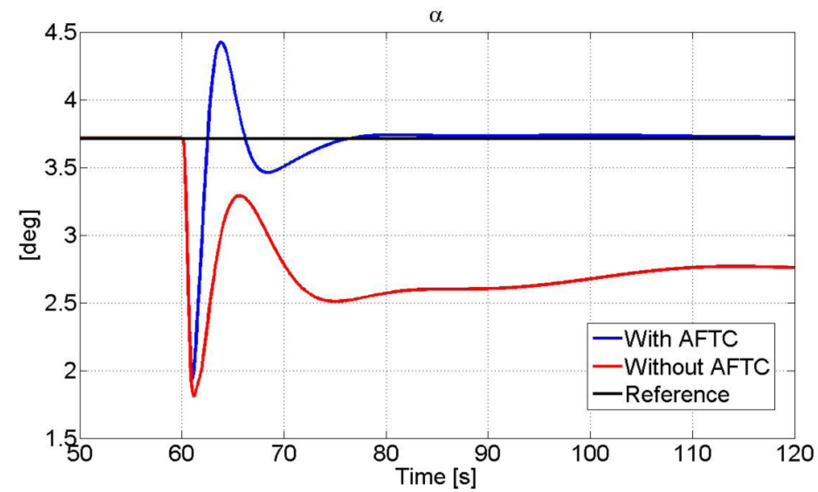


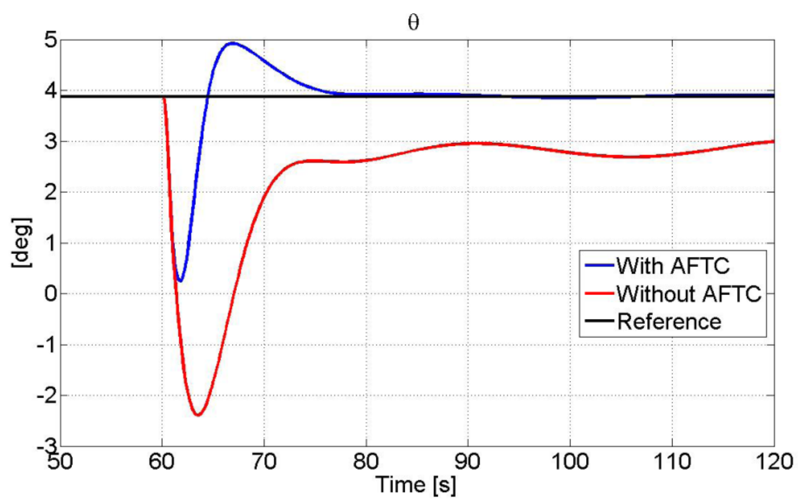


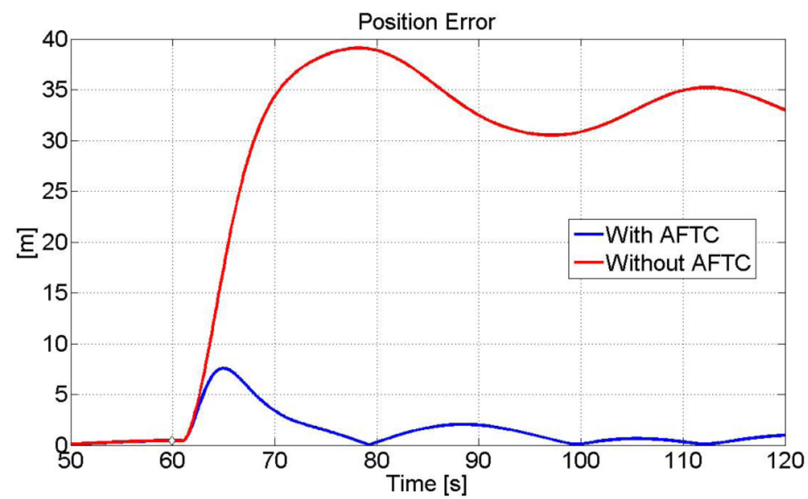
Case study : no wind condition, fault on elevator $f_{\delta_e} = +3^\circ$

red :with AFTCS **blue: without AFTCS**



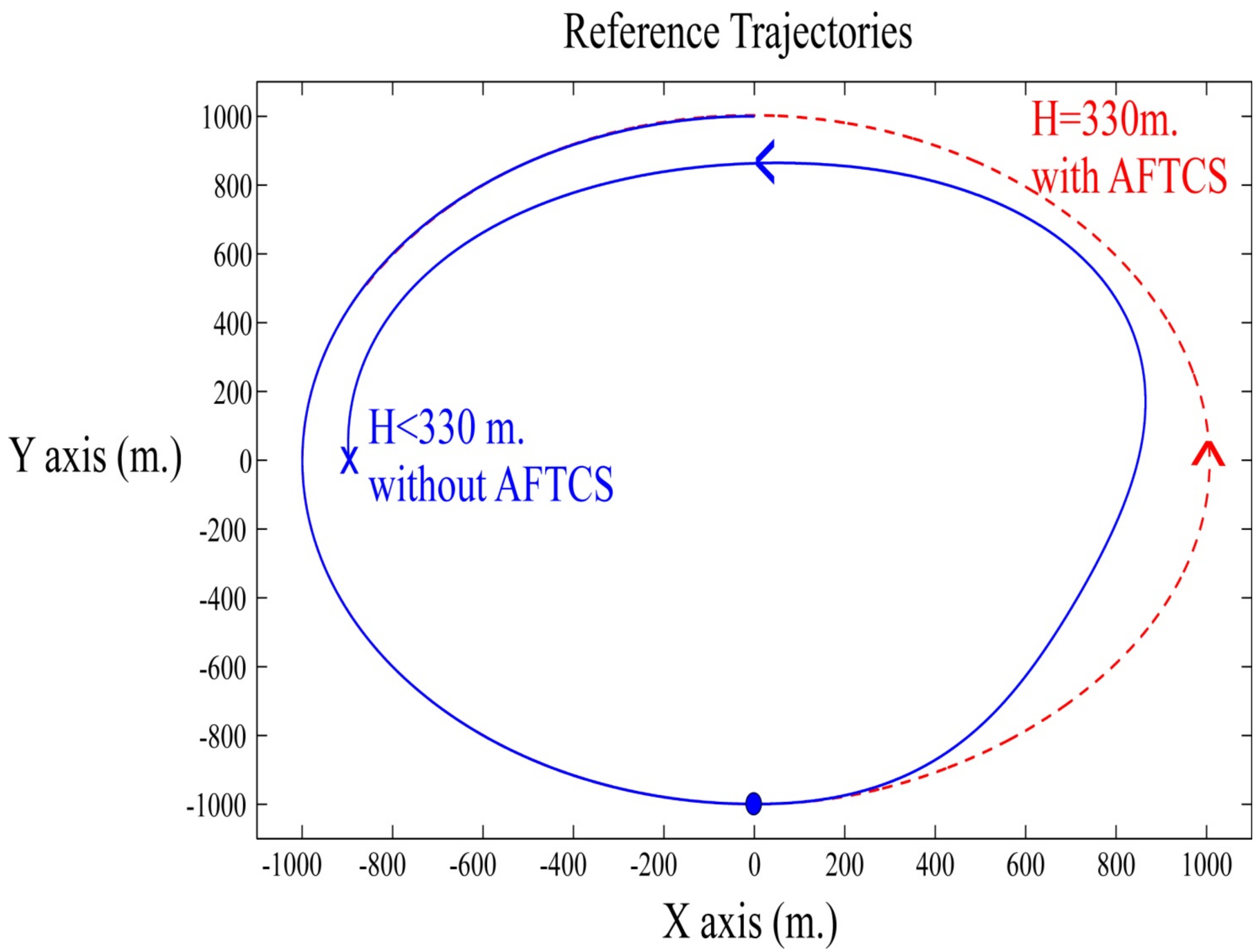






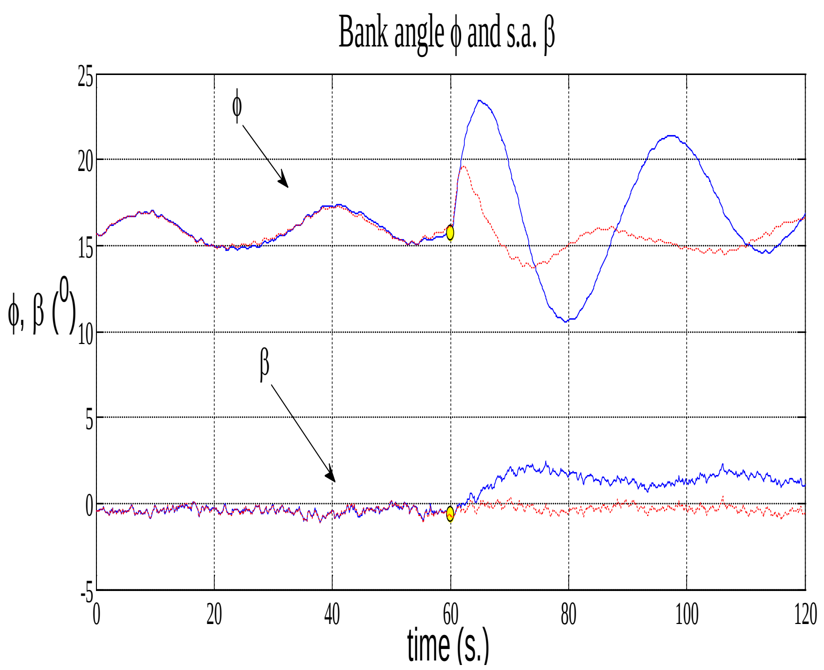
Case study : no wind condition, fault on aileron $f_{\delta_a} = -5^\circ$

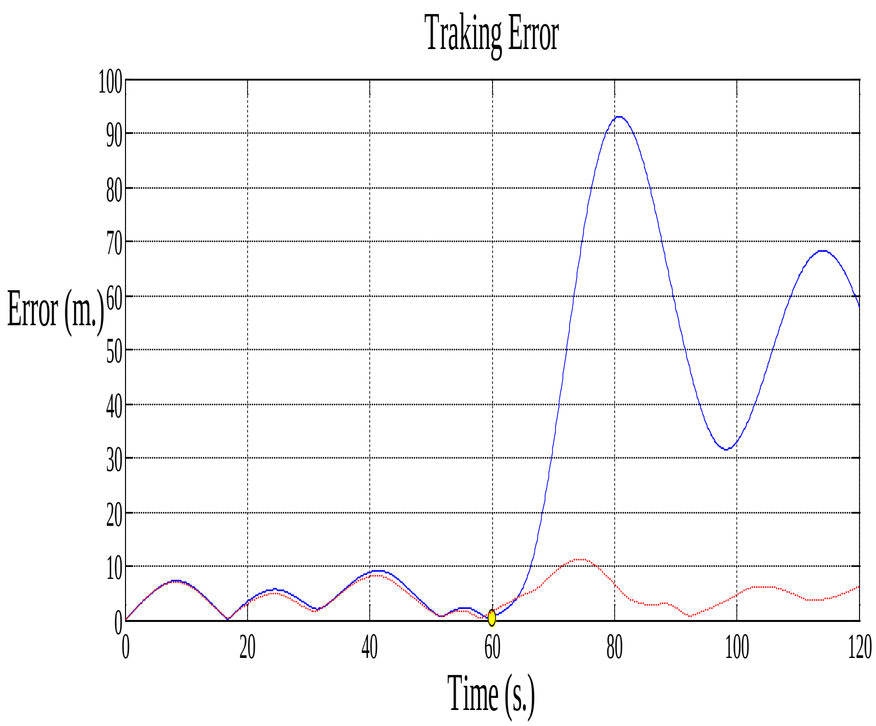
red :with AFTCS **blue: without AFTCS**



Case Study : with wind and measurements noises: fault on aileron $f_{\delta_a} = -5^{\circ}$

red :with AFTCS **blue: without AFTCS**





Recent advances of AFTC scheme

In

- **P. Castaldi, N. Mimmo and S. Simani, "Differential geometry based active fault tolerant control for aircraft" . Control Engineering Practice 2014. http://dx.doi.org/10.1016/j.conengprac.2013.12.011**
    - By means of the **singular perturbation (SP) theory** a differential geometry based controller has been designed for fast and slow aircraft dynamics
    - **The fault estimate convergence and the stability of the overall AFTCS controller are theoretically proved.**
    - **High fidelity simulations show the effectiveness of the scheme.**

In:

- **P Castaldi, N Mimmo and S Simani, " NonLinear Fault Tolerant Flight Control for Generic Actuators Fault Models". Proceedings of American Control Conference 2014, pagg. 1267-1272.**
    - **Estimate of actuator faults with generic time behaviour is proposed by exploiting Radial Basis Functions Neural Networks (RBF)**
    - **RBF allow approximating generic faults since do not need any a priori information about the fault internal model**

Remark: SP and RBF NN theory will be subject of future workshop

Other AFTC scheme in aerospace application: satellite

Satellite AFTCS

- P. Baldi, P. Castaldi, N. Mimmo and S. Simani, "Satellite Attitude Active FTC Based On
- Geometric Approach and RBF Neural Network" . 2013 Conference on Control and Fault-Tolerant Systems (SysTol) October 9-11, 2013. Nice, France
    - AFTCS for satellite (attitude control) in case of reaction wheels (actuator) faults
    - Generic faults estimates by means of RBF
    - Generalized bank of residuals for FDI then estimation of isolated fault by RBF
    - High fidelity simulations show the effectiveness of the scheme.

Satellite FDIR (Fault Detection, Isolation and controller Riconfiguration)

- P. Baldi, P Castaldi, N Mimmo and S Simani, "A new aerodynamic decoupled frequential FDIR methodology for satellite actuator faults". Int. J. Adapt. Control Signal Process. (2013) Published online in Wiley Online Library (wileyonlinelibrary.com).
  DOI: 10.1002/acs.2379

    - Estimate of reaction wheels actuator faults by means of a frequential approach
    - Reconfiguration of the controller by means of FDI diagnostic information

    - Remark: only aircraft case is given in this workshop due to time limit

A Longitudinal Flight Path Control with Adaptive Wind Shear Estimation and Compensation

- **An novel approach to the Longitudinal Guidance and Control (LGC) issue for an aircraft in presence of wind shear. WIND SHEAR COMPONENTS CONSIDERED AS FAULTS**

- **Adaptive estimation of the wind shear disturbances** : **design, based on the NonLinear Geometric Approach** state space coordinate change, of independent adaptive filters

- **Wind Shear components estimate exploited, in an original way, by a BackStepping based controller**, thus resulting in an Adaptive BackStepping Controller

- **Test of the Longitudinal Guidance and Control system**
  - **detailed flight simulator implementing the real wind shear condition which caused the 1975 crash of Eastern Flight 066 at JFK airport**
  - **Monte Carlo robustness test**

LGC systems and Wind Shear: state of art

- **Wind Shear** ⇒ spatial and temporal abrupt change of wind speed and direction
  ⇒ significant and potentially catastrophic hazard to aircraft taking off or landing.

- **Last decades** ⇒ developing of accurate *Wind Shear models* **exploiting mathematical functions (e.g. sinusoidal functions) or real data observations.**

  ⇒ **proper *control strategies*: mainly robust control methods** allowing the aircraft to complete safe abort-landing or take off procedures

  ⇒ **these methods do not permit to complete precision procedures with adequate safety level and desired performance.**

- ***Existing Wind Shear estimation methods*** are based on Extended **Kalman Filter** (EKF) and Unscented Kalman Filters (UKF) (see e.g. Stengel, Zhou)

  ⇒ **differently from our proposed estimation scheme they exploit vertical velocity and acceleration measurements** in addition to position and attitude measurements. Moreover they have large dimensions and heavy computational burden

Proposed LGC systems

- Development of a **Longitudinal Guidance and Control (LGC)** system which allows the aircraft to follow the glide slope trajectory with constant speed during a precision approach ***in presence of Wind Shear***
  - **based on** the design of **three independent Adaptive Filters** providing the **estimates of the wind shear disturbance components**.
  - **estimates exploited in a Backstepping - based controller**
    - **Adaptive BackStepping control scheme for compensating wind shear effects.**
- **Note**: **Adaptive Filters based on the Nonlinear Geometric Approach**
  - **exploit only attitude and height measures**
  - **allow to estimate and compensate wind shear effects in a *quick* and effective manner.**

References

The approach described in the following can be found in

P. Baldi, P. Castaldi, N. Mimmo, A. Torre and S. Simani, A New Longitudinal Flight Path Control with Adaptive Wind Shear, Estimation and Compensation2011 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC) Orlando, FL, USA, December 12-15, 2011

Recent advances of the approach in

- **P. Baldi, P. Castaldi, N Mimmo, S Simani , Generic Wind Estimation and Compensation Based on NLGA and RBF-NN P Baldi, Proceedings 2014 European Control Conference , June 24-27, 2014. Strasbourg, France**

  - **Time varying diffeomorphism theory to exploit wind estimate in the controller design (based on Feedback-Linearization and Backstepping)**
  - **Wind shear components estimate obtained by means of RBF-NN based adaptive filters**

Remark: Time varying diffeomorphism theory in future workshop

Aircraft Dynamic Model (with Wind Shear)

Aircraft equations of motion

$$\begin{cases} \dot{V} = \dfrac{1}{m}\left[T\cos\alpha - D - mg\sin\gamma - m\left(\dot{W}_x\cos\gamma + \dot{W}_h\sin\gamma\right)\right] \\ \dot{\gamma} = \dfrac{1}{mV}\left[T\sin\alpha + L - mg\cos\gamma + m\left(\dot{W}_x\sin\gamma - \dot{W}_h\cos\gamma\right)\right] \\ \dot{\alpha} = q - \dot{\gamma} \\ \dot{q} = M / I_y \end{cases}$$

$x = \left[V, \gamma, \alpha, q\right]^T$ airspeed, flight path angle, angle of attack, pitch rate;

m aircraft mass;

$\dot{W}_x, \dot{W}_h$: **derivative of wind shear horizontal and vertical components**

Aerodynamic model

The aerodynamic effects are Lift and Drag forces L ,D and pitching moment M

These effects can be expressed in term of aerodynamic coefficients:

$$\begin{cases} D = \overline{q}SC_D = \overline{q}S\left(C_{D0} + C_{D\alpha}\alpha + C_{D\delta_e}\delta_e\right) \\ L = \overline{q}SC_L = \overline{q}S\left(C_{L0} + C_{L\alpha}\alpha + C_{Lq}\hat{q} + C_{L\delta_e}\delta_e\right) \\ M = \overline{q}S\overline{c}C_m = \overline{q}S\overline{c}\left(C_{m0} + C_{m\alpha}\alpha + C_{mq}\hat{q} + C_{m\delta_e}\delta_e\right) \end{cases}$$

$\overline{q} = \rho V^2 / 2$ dynamic pressure ; ρ air density ;

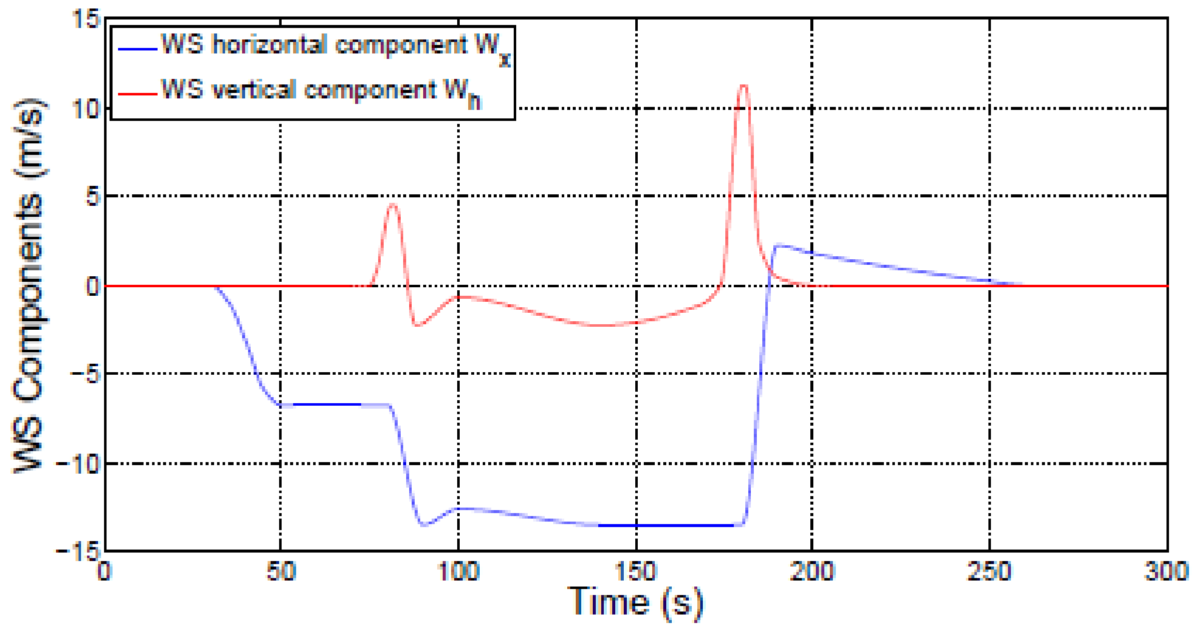
$\hat{q} = q\overline{c} / (2V)$ dimensionless pitch rate

S wing surface; $\overline{c}$ mean aerodynamic chord ;

$u = \left[T, \delta_e\right]^T$ **are the thrust and the elevator command respectively (inputs).**

Wind Shear Model : vertical and horizontal components

Reconstruction of data referring to the crash of a Boeing 727 occurred on 24 June 1975 (JFK , New York).



W_x **,** W_h**: example of the worst conditions for an aircraft**

Valid benchmark for testing the validity of the proposed method

Assumption 1

The wind shear components can be approximated by their trapezoidal envelops: their first time derivatives, $\dot{W}_x$ ***and*** $\dot{W}_h$ ***are piecewise constant functions (trapezoidal approximation)***

Wind Shear estimation as an FDD problem

Use of a coordinate change in state space (De Persis and Isidori) in order **to find an observable subsystems affected by a fault and not affected by disturbances** (other fault to be decoupled).

It is assumed that the wind shear derivative components , $\dot{W}_x$ and $\dot{W}_h$, can be alternatively viewed as faults, to be estimated, acting on the system and have to be reciprocally decoupled.

Two subsystems are determined:

- **the first is affected by $\dot{W}_h$ and decoupled from $\dot{W}_x$**
- **the second is affected by $\dot{W}_x$ and decoupled from $\dot{W}_h$**

The design of Adaptive Filters estimating Wind Shear derivative signals are based on these subsystems

A third adaptive filter, provide the estimate of the wind shear vertical component W_h which affects the aircraft rate of climb .

$\dot{W}_h, \dot{W}_x, W_h$ estimate: subsystems

For the estimation of the two wind shear components derivative $\dot{W}_x$ and $\dot{W}_h$ it is possible to find two new independent variables:

1) $\boxed{\bar{x}_1 = V\cos\gamma}$ $\Rightarrow$ $\bar{x}_1$-subsystem $\dot{\bar{x}}_1 = \dot{V}\cos\gamma - V\dot{\gamma}\sin\gamma$
affected by $f = \dot{W}_x$ decoupled from $d = \dot{W}_h$

2) $\boxed{\bar{x}_1 = V\sin\gamma}$ $\Rightarrow$ $\bar{x}_1$ - subsystem $\dot{\bar{x}}_1 = \dot{V}\sin\gamma + V\dot{\gamma}\cos\gamma$
affected by $f = \dot{W}_h$ decoupled from $d = \dot{W}_x$

W_h obtained by a numerical integration of its derivative estimation :

- **integrated quantity differs from the true wind shear component by a constant bias**
- to estimate bias a further filter it is based on the subsystem

$\boxed{\bar{x}_1 = H}$ $\Rightarrow$ $\bar{x}_1$ - subsystem $\dot{\bar{x}}_1 = \dot{H} = V\sin\gamma + W_h$

$\Rightarrow$ ***three independent and independently trimmable estimation filters***

NLGA- Adaptive Filters for $\dot{W}_x, \dot{W}_h, W_h$

For the first adaptive filter, which is **sensitive only to** $f = \dot{W}_x$ we have

$$\begin{cases} \dot{\bar{y}}_{1s}(t) = M_1(t)\cdot\dot{W}_x + M_2(t) \quad \text{with } M_1(t) = -1 \\ M_2(t) = \dfrac{T}{m}\left[\cos\gamma\cos\alpha - \sin\gamma\sin\alpha\right] - \dfrac{\bar{q}S}{m}\left[C_D\cos\gamma + C_L\sin\gamma\right] \end{cases}$$

For the second adaptive filter, which is **sensitive only to** $f = \dot{W}_h$ we have

$$\begin{cases} \dot{\bar{y}}_{1s}(t) = M_1(t)\cdot\dot{W}_h + M_2(t) \quad \text{with } M_1(t) = -1 \\ M_2(t) = \dfrac{T}{m}\left[\sin\gamma\cos\alpha + \cos\gamma\sin\alpha\right] - \dfrac{\bar{q}S}{m}\left[C_D\sin\gamma - C_L\cos\gamma\right] - g \end{cases}$$

Finally, the third filter, estimating W_h, is based on

$$\begin{cases} \dot{\bar{y}}_{1s}(t) = M_1(t)\cdot W_{hBIAS} + M_2(t) \quad \text{with } M_1(t) = 1 \\ M_2(t) = V\sin\gamma + \int_0^t \hat{\dot{W}}_h(t)\,dt \end{cases}$$

The three adaptive are completely independent and independently trimmable.
They exploit only attitude and height measures resulting fast and accurate as shown in simulation results

BackStepping control signals: inner loop

- **Distance d between the aircraft and the reference is precisely provided by Instrument Landing System (ILS).**

- Since to follow the reference path the aircraft has to maintain a certain Rate of Climb $RoC = \dot{H} = V \sin\gamma$ ⟹ developing of both airspeed and flight path angle control . From inner loop to outer loop as follows:

Backstepping control signals, inner loop: the pitch rate equation can be written as

$$\dot{q} = f_q + g_q \delta_e \quad \text{with} \quad f_q = \frac{\overline{q} S \overline{c}}{I_y}\left(C_{m0} + C_{m\alpha}\alpha + C_{mq}\hat{q}\right); \quad g_q = \frac{\overline{q} S \overline{c}}{I_y} C_{m\delta_e} \neq 0$$

Inverting previous equation, we obtain the elevator control signal resulting in the desired dynamics of the pitch rate $\dot{q}_c$ ⟹ **Elevator control signal**

$$\boxed{\delta_e = \frac{1}{C_{m\delta_e}}\left[-C_{m0} - C_{m\alpha}\alpha - C_{mq}\hat{q} + \dot{q}_c \frac{I_y}{\overline{q} S \overline{c}}\right]}$$

Backstepping control signals: outer loop

The dynamic inversion algorithm requires the simplification of the model
Some aerodynamic contribution are neglected ⟹ **Synthesis model:**

$$\begin{cases} \dot{V} = T\dfrac{\cos\alpha}{m} - \dfrac{\overline{q}S}{m}\left(C_{D0} + C_{D\alpha}\alpha\right) - g\sin\gamma \\ \dot{\gamma} = \dfrac{1}{V}\left[\dfrac{T}{m}\sin\alpha + \dfrac{\overline{q}S}{m}\left(C_{L0} + C_{L\alpha}\alpha\right) - g\cos\gamma\right] \\ \dot{\alpha} = q - \dfrac{1}{V}\left[\dfrac{T}{m}\sin\alpha + \dfrac{\overline{q}S}{m}\left(C_{L0} + C_{L\alpha}\alpha + C_{Lq}\hat{q}\right) - g\cos\gamma\right] \end{cases}$$

Finally, from previous equation, **we obtain the outer loop backstepping control signals as follows** ⟹ **see next slide**

Backstepping control signals: outer loop (2)

By means of Dynamic Inversion (DI) , it is obtained

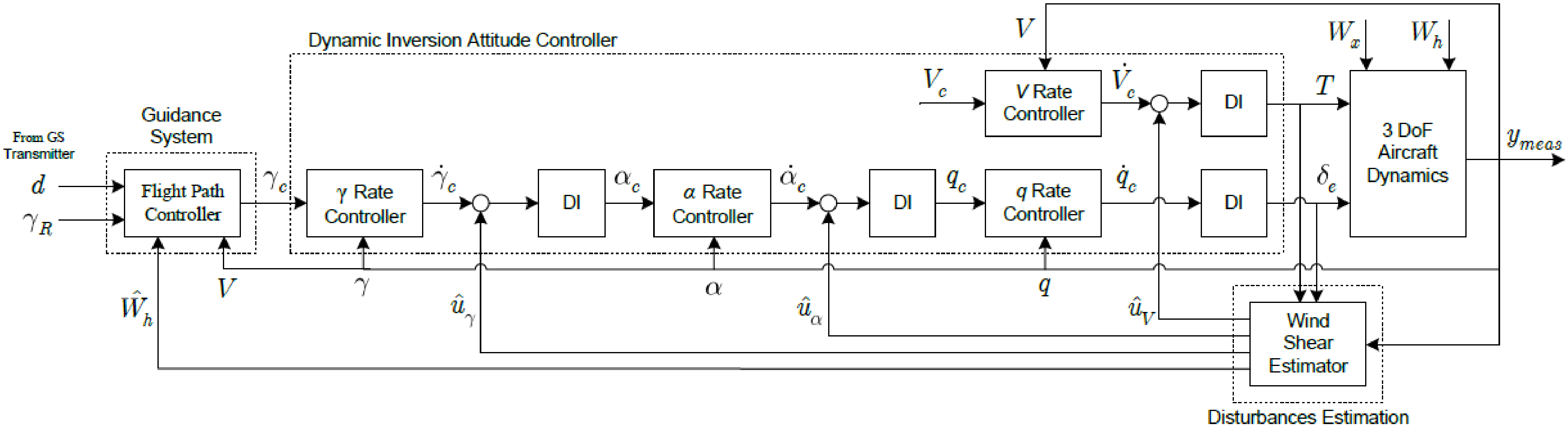
$$\begin{cases} q_c = \dot{\alpha}_c + \dfrac{\bar{q}S}{mV}\left(C_{L0} + C_{L\alpha}\alpha + C_{Lq}\hat{q}\right) + T\dfrac{\sin\alpha}{mV} - \dfrac{g\cos\gamma}{V} \\ \alpha_c = \dfrac{1}{C_{L\alpha}}\left[-C_{L0} + \dfrac{mV}{\bar{q}S}\left(\dot{\gamma}_c - T\dfrac{\sin\alpha}{mV} + \dfrac{g\cos\gamma}{V}\right)\right] \\ T = \dfrac{1}{\cos\alpha}\left[\bar{q}S\left(C_{D0} + C_{D\alpha}\alpha\right) + mg\sin\gamma + m\dot{V}_c\right] \end{cases}$$

The desired signal $\dot{\alpha}_c, \dot{\gamma}_c, \dot{V}_c$, denoted with $\dot{y}_c$, can be practically implemented by means of a PI controller (see Snell, 1992 et al; Farrel et al. 2003 for further methods)

$$\dot{y}_c = K_p(y_c - y) + K_i\int_0^t (y_c - y)dt$$

where the gain coefficients are determined by applying a linear theory

Backstepping Guidance and Control System structure



•Flight Path Angle Autopilot and Airspeed Autopilot: $\boxed{\gamma_c(t), V_c(t) \Rightarrow T(t), \delta_e(t)}$

V_c = constant

$\gamma_c(t)$ **is a function of the aircraft distance from the glide path**

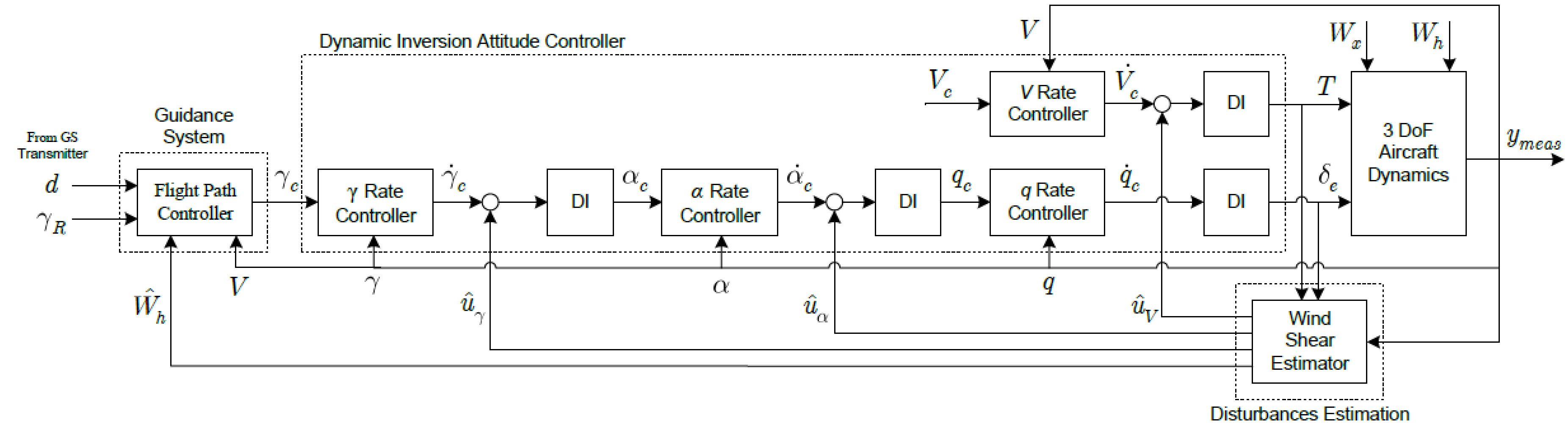
$$\gamma_c = \arcsin\frac{\dot{d}_c}{V}; \quad \dot{d}_c = -K_{pd}d - K_{id}\int_0^t d\,dt$$

The aircraft is maintained at a zero-distance from the glide path with constant speed

Correction signals $\hat{W}_h(t), \hat{u}_V, \hat{u}_\gamma, \hat{u}_\alpha$

based on Vertical Wind Shear and Wind Shear Derivative component estimates

Correction Signal based on Wind Shear estimate



$$\dot{H} = V \sin\gamma + W_h \Rightarrow \gamma = \arcsin\frac{\dot{H} - W_h}{V} \Rightarrow \boxed{\gamma_c = \arcsin\frac{\dot{d}_c - \hat{W}_h}{V}}$$

$\gamma_c(t)$ **is obtained by means of two contributions:**

•the first one (glide slope autopilot) make use of d distance measure provided by ILS

•second exploits the estimate $\hat{W}_h$ provided by the adaptive filter, in order to compensate Wind Shear effects in real-time

$\hat{u}_V, \hat{u}_\gamma, \hat{u}_\alpha$ **correction signals based on feedback of the Wind Shear derivative estimates** ⟹ **see next slide**

Correction signal based on Wind Shear estimate (2)

The desired signals $\dot{V}_c$, $\dot{\gamma}_c$ and $\dot{\alpha}_c$ are corrected through the feedback of the wind shear derivative estimates $\hat{\dot{W}}_x(t)$ and $\hat{\dot{W}}_h(t)$

$$\begin{cases} \hat{u}_V = -\hat{\dot{W}}_x \cos\gamma - \hat{\dot{W}}_h \sin\gamma \\ \hat{u}_\gamma = \dfrac{1}{V}\left(\hat{\dot{W}}_x \sin\gamma - \hat{\dot{W}}_h \cos\gamma\right) \\ \hat{u}_\alpha = -\hat{u}_\gamma \end{cases}$$

the final formulation for the desired output signals as follows:

$$\begin{cases} \dot{V}_c = K_{pV}(V_c - V) + K_{iV}\int_0^t (V_c - V)\,dt - \hat{u}_V \\ \dot{\gamma}_c = K_{p\gamma}(\gamma_c - \gamma) + K_{i\gamma}\int_0^t (\gamma_c - \gamma)\,dt + \hat{u}_\gamma \\ \dot{\alpha}_c = K_{p\alpha}(\alpha_c - \alpha) + K_{i\alpha}\int_0^t (\alpha_c - \alpha)\,dt - \hat{u}_a \end{cases}$$

Simulation Results

Performances of Adaptive Filters and the guidance and control scheme:

a RQ-2 Pioneer UAV simulator has been used.

TABLE I

SIMULATION PARAMETERS

$m = 190.512$ [kg]	$C_{D0} = 0.060$
$I_y = 90.948$ [kg m^2]	$C_{L0} = 0.385$
$S = 2.826$ [m^2]	$C_{m0} = 0.194$
$\bar{c} = 0.548$ [m]	$C_{D\alpha} = 0.430$ [rad^{-1}]
$H_0 = 500$ [m]	$C_{D\delta_e} = 0.018$ [rad^{-1}]
$\rho = 1.168$ [kg m^{-3}]	$C_{L\alpha} = 4.780$ [rad^{-1}]
$g = 9.779$ [m s^2]	$C_{L\delta_e} = 0.410$ [rad^{-1}]
$V_0 = 35$ [m s^{-1}]	$C_{Lq} = 8.050$ [rad^{-1}]
$\gamma_R = -3$ [deg]	$C_{m\alpha} = -2.120$ [rad^{-1}]
$T_{max} = 667.230$ [N]	$C_{m\delta_e} = -1.760$ [rad^{-1}]
$\delta_{e_{max}} = +20$ [deg]	$C_{mq} = -36.600$ [rad^{-1}]
$\delta_{e_{min}} = -20$ [deg]	$t_{sim} = 300$ [s]

TABLE II

SIMULATED SENSOR NOISES

$3\sigma_V = 1.5$ [m/s]	$3\sigma_\gamma = 1.5$ [deg]
$3\sigma_\alpha = 1.0$ [deg]	$3\sigma_q = 1.0$ [deg/s]
$3\sigma_H = 1.0$ [m]	$3\sigma_T = 10$ [N]
$3\sigma_{\delta_e} = 1.0$ [deg]	

Table I: geometric and aerodynamic characteristics of the aircraft and the simulation parameters

Assumptions:

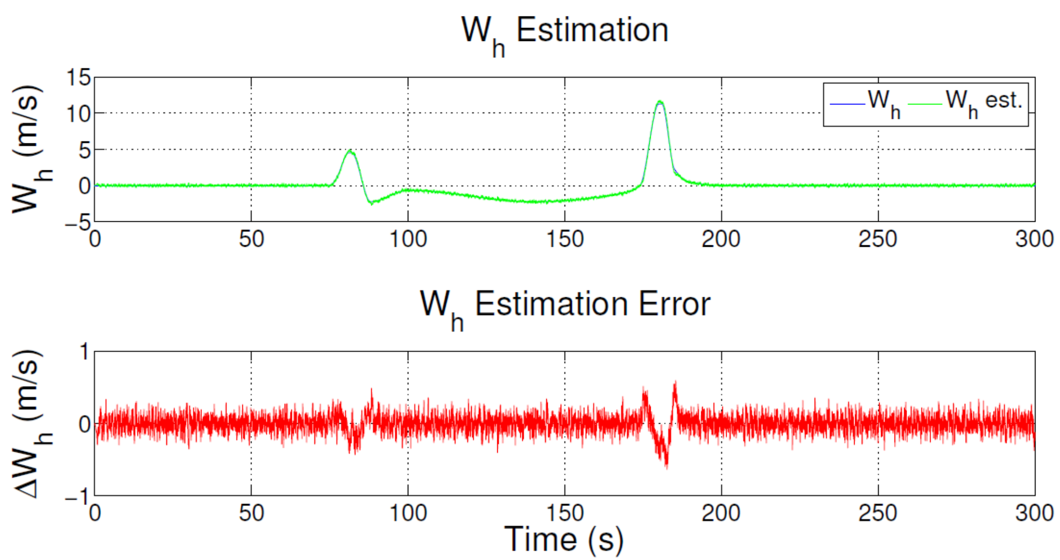
- the aircraft mass is constant
- the air density is constant
- **a first order dynamics has been introduced to simulate the engine response**
- **input and output sensors are characterized by Gaussian additive white noises(Table II: standard deviations)**

Wind Shear Estimate

- **The designed bank of adaptive filters demonstrates good transient response and provides asymptotic unbiased estimates of** $\boxed{\hat{\dot{W}}_x(t),\ \hat{\dot{W}}_h(t)\ ,\ \hat{W}_h(t)}$

- **Estimation filters robust under working condition (actual wind) that differs from the design assumptions (trapezoidal wind, i.e. wind with constant derivative).**

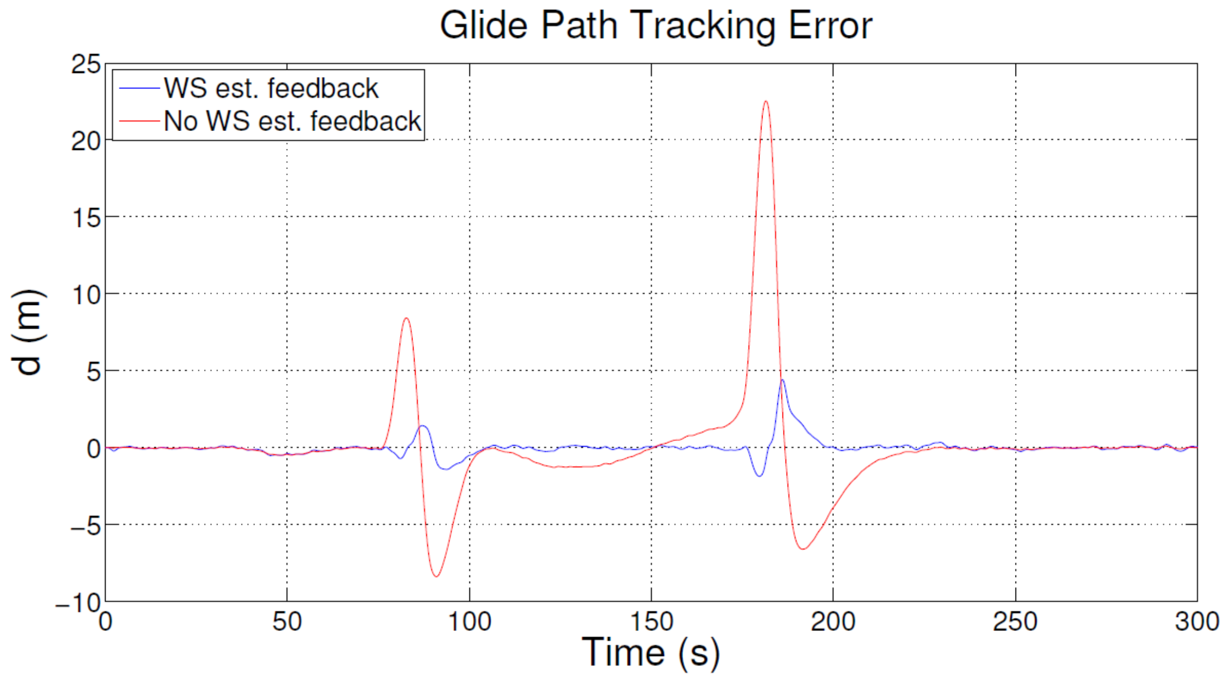
- **The figure shows the wind shear vertical component estimate in comparison with its actual values. The estimation error is given in the lower subplot.**



Similar results were obtained for the estimated wind shear derivative.

Tracking error with and without WS estimate

Glide Path Tracking error d for the Guidance and Control System (GCS) provided or not with the wind shear estimates



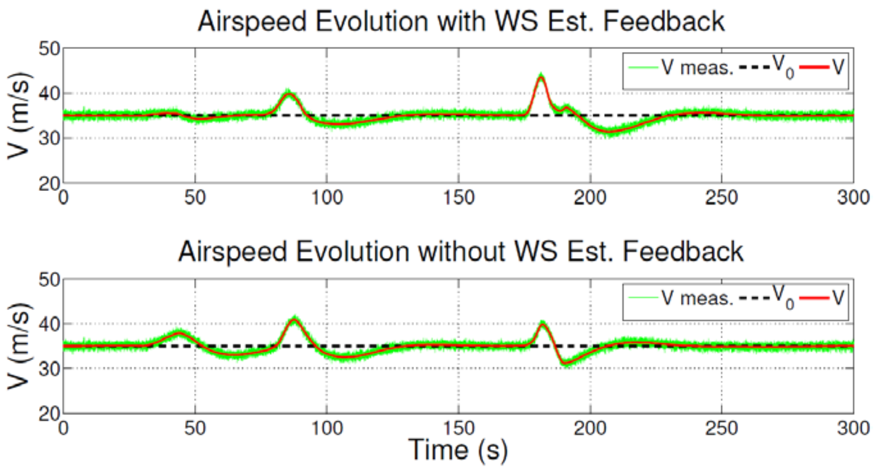
Deviation higher in the case of a GCS without the compensation of the wind shear effects, even if the guidance system is capable to restore the right descent trajectory and annihilates the tracking error in any case.

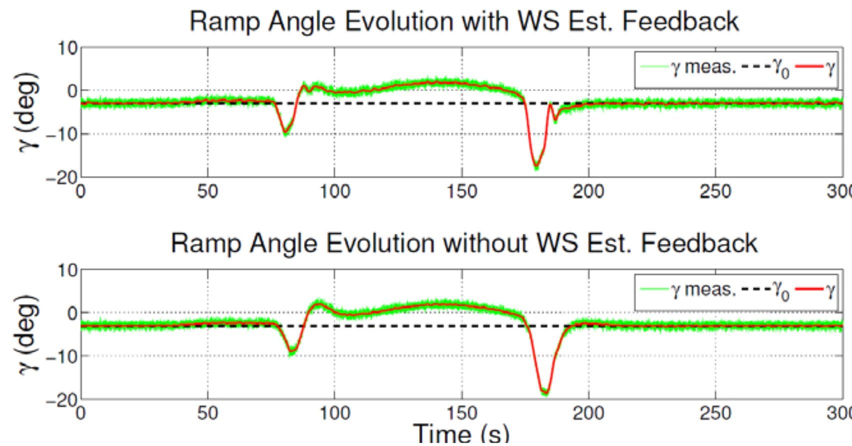
GCS without the wind shear compensation: Maximum Tracking error about 22.5 meters

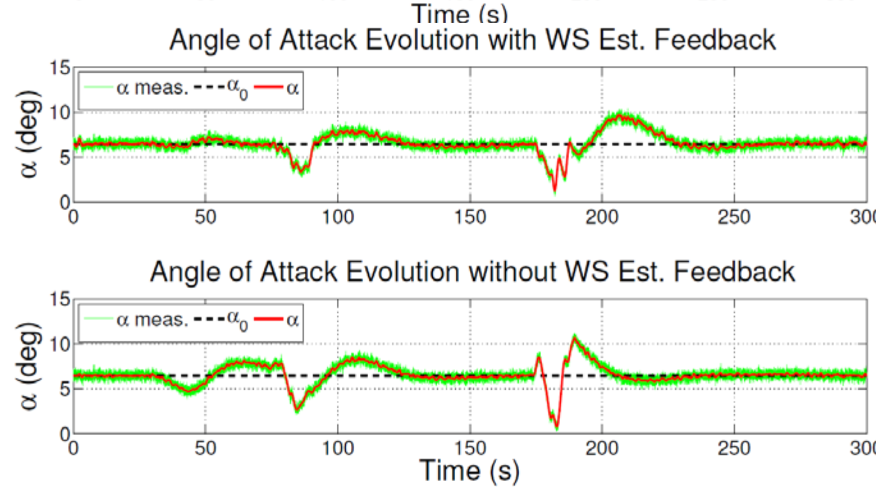
GCS with the wind shear compensation: Maximum Tracking error about 4.5 meters

V, γ and α with and without WS compensation

The figures highlight also the differences between the actual output of the system and the measured output. The latter is clearly deteriorated by the simulated sensor noises and affects both the AF estimates and the actual state itself, since it is used for control purpose.







- **Angle of attack < 10° and the airspeed > 32 m/s.** This values are **far enough from the stall condition**, **which is at about 13° for an airspeed of 27 m/s.**
- **A substantial improvement of angle of attack, in presence of a better tracking error!**
- **Moreover the thrust and the elevator angle values never exceeded the operative limits**

Conclusion

- **Novel AFTCs based on NLGA applied to aircraft 6dof model and detailed simulator**
  - **NLGA -> subsystems affected by fault and decoupled from disturbances (wind) and other faults**
  - **Adaptive Filters providing fault estimates**
  - **Further Feedback loop exploiting detected and isolated fault estimate:**
    Built-in controller can be maintained
  - **Recent advances**
    - **Singular perturbation theory: fast and slow aircraft dynamic based controller and FDI**
    - **RBF-NN: generic fault estimate**
    - **Analytical proof of convergence of the fault estimate and stability of the overall control system**
- **Novel longitudinal guidance and control system, for an aircraft affected by wind shear disturbances (considered as fault)**
  - Development of three independently designed adaptive filters which provide the estimate of the **wind shear disturbance components viewed as faults**
  - **Wind Shear estimate exploited for BackStepping based control signals**, thus resulting in an Adaptive BackStepping Control Scheme
  - **wind shear data from a simulated actual crash situation**
  - **Recent advances: RBF-NN and time varying diffeomorphism -> differential geometry based overall control system in case of generic wind disturbances**

Application of AFTCs for generic systems: (recommended reading of AMCS)

Marcello Bonfè, Paolo Castaldi, Nicola Mimmo, Silvio Simani, Active fault tolerant control of nonlinear systems: The cart-pole example. International Journal of Applied Mathematics and Computer Science (AMCS) 2011, Vol. 21,No. 3

More References

Books:

M. Benini, P. Castaldi, S. Simani, Fault Diagnosis for Aircraft System Models Number of pages: 159, ISBN: 978-3-639-21364-5, Publisher: VDM Verlag Dr. Muller Aktiengesellshaft & Co. KG, Date: 2009
(where also an introducuction to FDI and linear methods for aircraft FDI are presented)

Nader Meskin,Khashayar Khorasani, Fault Detection and Isolation: Multi-Vehicle Unmanned Systems, Elsevier
(where also fault on redundant actuators are presented)

Lectures:

G. Bertoni, P.Castaldi, N. Bertozzi, M. Bonfè, S. Simani. “Guidance and Nonlinear Active Fault Tolerant Control for General Aviation Aircraft”. IFAC Workshop on Aerospace Guidance, Navigation and Flight Control Systems AGNFCS'09 Samara, RUSSIA , Plenary Lecture, June 30 - July 2, 2009 (also mentioned on newsletter IFAC)

M. Bonfè,P. Castaldi , S. Simani , S. Beghelli.“Integrated Fault Diagnosis and Fault Tolerant Control Design for an Aircraft Model. International Conference [on Systems Engineering (ICSE), Plenary Lecture , 8-10 September 2009