Overview

Synthesis of SISO Controllers

Pole Assignment

In the previous chapter, we examined PID control. However, the tuning methods we used were essentially ad-hoc. Here we begin to look at more formal methods for control system design. In particular, we examine the following key synthesis question:

> *Given a model, can one systematically synthesize a controller such that the closed loop poles are in predefined locations*?

This lecture will show that this is indeed possible. We call this *pole assignment*, which is a fundamental idea in control synthesis.

Polynomial Approach

In the nominal control loop, let the controller and nominal model transfer functions be respectively given by:

$$C(s) = \frac{P(s)}{L(s)} \qquad G_o(s) = \frac{B_o(s)}{A_o(s)}$$

with

$$\begin{aligned} P(s) &= p_{n_p} s^{n_p} + p_{n_p-1} s^{n_p-1} + \ldots + p_0 \\ L(s) &= l_{n_l} s^{n_l} + l_{n_l-1} s^{n_l-1} + \ldots + l_0 \\ B_o(s) &= b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \ldots + b_0 \\ A_o(s) &= a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0 \end{aligned}$$

Consider now a desired closed loop polynomial given by

$$A_{cl}(s) = a^c_{n_c} s^{n_c} + a^c_{n_c-1} s^{n_c-1} + \ldots + a^c_0$$

Goal

Our objective here will be to see if, for given values of B_0 and A_0, P and L can be designed so that the closed loop characteristic polynomial is A_{cl}(s).

We will see that, under quite general conditions, this is indeed possible.

Lemma 7.1: (SISO pole placement. Polynomial approach). Consider a one d.o.f. feedback loop with controller and plant nominal model given above. Assume that $B_0(s)$ and $A_0(s)$ are relatively prime (coprime), i.e. they have no common factors. Let $A_{cl}(s)$ be an arbitrary polynomial of degree $n_c = 2n - 1$. Then there exist polynomials $P(s)$ and $L(s)$, with degrees $n_p = n_l = n - 1$ such that

$$A_o(s)L(s) + B_o(s)P(s) = A_{cl}(s)$$

PI and PID Synthesis Revisited using Pole Assignment

The reader will recall that PI and PID controller synthesis using classical methods were reviewed in Lecture 6.

During laboratory sessions we place these results in a more modern setting by discussing the synthesis of lead/lag networks and PID controllers based on pole assignment techniques (via root locus analysis).

Summary

- This chapter addresses the question of synthesis and asks: *Given the model $G_0(s) = B_0(s)/A_0(s)$, how can one synthesize a controller, $C(s) = P(s)/L(s)$ such that the closed loop has a particular property.*
- Recall:
  - the poles have a profound impact on the dynamics of a transfer function;
  - the poles governing the closed loop belong to the same set, namely the roots of the characteristic equation $A_0(s)L(s) + B_0(s)P(s) = 0$.

- Therefore, a key synthesis question is:
  Given a model, can one synthesize a controller such that the closed loop poles (i.e. sensitivity poles) are in pre-defined locations.
- Stated mathematically:
  Given polynomials $A_0(s)$, $B_0(s)$ (defining the model) and given a polynomial $A_{cl}(s)$ (defining the desired location of closed loop poles), is it possible to find polynomials $P(s)$ and $L(s)$ such that $A_0(s)L(s) + B_0(s)P(s) = A_{cl}(s)$? This lecture shows that this is indeed possible.